MATH 230 Homework 2

Due to April 12, 2018, before 17:40

NAME/SURNAME:..

STUDENT NO:..

SECTION NO:............................ DEPARTMENT:............................

IMPORTANT

1 Check that there are 8 questions on your booklet. Not all questions will be graded.

2 Show all your work. Correct answers without sufficient explanation may not get full credit.

3 Write your name on each page.

Q?	Q?	Q?	Q?	Q?	TOTAL
4	4	4	4	4	20

Problem 1. The Rockwell hardness of a metal specimen is determined by impressing the surface of the specimen with a hardened point, and then measuring the depth of penetration. The hardness of a certain alloy is normally distributed with a mean of 70 units and standard deviation of 3 units.

(a) If a specimen is acceptable only if its hardness is between 66 and 74 units, what is the probability that a randomly chosen specimen is acceptable?

(b) If the acceptable range is $70 \pm c$, for what value of c would 95% of all specimens be acceptable?

Problem 2. One thousand independent rolls of a fair die will be made.

(a) Find an approximate value of the probability that number 6 will appear between 150 and 200 times inclusively.

(b) If number 6 appears exactly 200 times, estimate the probability that number 5 will appear less than 150 times.

Problem 3. Let X be an exponential random variable with mean 2. Let $Y = X^5$.

(a) Write the probability density function of random variable Y.

(b) Find the expected value and the variance for Y.

Problem 4. Let X be a normal random variable with parameters μ and σ ($\sigma > 0$), that is the density function of X is given as

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

Show that (a) $E[X] = \mu$ and (b) $V(X) = \sigma^2$.

Problem 5. A code word contains 6 digits: each either 0 or 1: to be valid the word must contain exactly four 1's and two 0's. One word is selected at random form the valid code words. Define X_1 to be the first (left most digit) and let X_2 be the second digit in the word selected.

(a) Find the joint probability distribution for X_1 and X_2 and marginal probabilities of X_1 and X_2.

(b) Find $cov(X_1, X_2)$.

Problem 6. The joint density of random variables $X$and Y is given by

$$f(x,y) = \begin{cases} Cxy, & 0 \le x \le 1,\ 0 \le y \le 1,\ 0 \le x+y \le 1, \\ 0, & otherwise. \end{cases}$$

(a) Find constant C.

(b) Find the marginal density of X.

(c) Find the marginal density of Y.

(d) Are X and Y independent? Explain.

(e) Find $E[X]$, $E[Y]$, $E[5X - 3Y]$.

(f) Find $P(X \le 0.5;\ Y \ge 0.75)$ and $P(X \ge 0.25;\ 0.25 \le Y \le 0.5)$.

Problem 7. Let $X_1, X_2, \cdots, X_n$ be independent random variables, and suppose that X_i has an exponential distribution with mean μ_i. Find the distribution of $M_n = \min(X_1, X_2, \cdots, X_n)$.

Problem 8. Let $U \sim$ Uniform(0,1). For $\alpha > 0$ and $\beta > 0$, determine the distribution of $T = (-\beta \log U)^{\alpha}$.