Assignment #1
Linear Programming (LP)
Formulation, Graphical Method and Excel Solver

ADM2302 students are reminded that submitted assignments must be neat, readable, and well-organized. Assignment marks will be adjusted for sloppiness, poor grammar and spelling, as well as for technical errors. While working together is encouraged, plagiarism on assignments will not be accepted. The assignment is to be submitted electronically as a single PDF file via blackboard learn by February 7^{th} prior to 23:59. Front page of the PDF document has to include title of the assignment, course code and section, student name and student number. Second page is *the individual statement of integrity that must be signed.*

Note: *Each student must provide an individual original submission of completed Assignment #1.* Please also note: Assignment #1 copies that are submitted jointly (i.e., by more than one author) will not be graded.

Problem 1 (15 points)

The price of oil has been dropping significantly recently. This urges fund managers to adjust their investment portfolios. In particular, many fund managers in Canada used to have a significant portion of their investments in the energy sector, but now they need to look into other investment opportunity. The fund manager of XYZ investment corp. estimates that in the past the rate-of-return for the investment made in the energy sector is about 3%, i.e. 100 dollars of investment can generate 3 dollars of profit, but given the trend of oil price the manager also predicts that the rate of return might drop to -5% in the worst-case scenario. The manager read the market reports of the other sectors and learned that in average the past rate-of-return for non-energy sectors is around 1.5%. Given that investing in other sectors appears to be less risky, the manager predicts that even in the worst-case scenario the rate-of-return for non-energy sectors can still be around 1%.

The manager needs to determine how much money she should invest in the energy sector and how much she should invest in the non-energy sectors so that the total profit estimated based on the past rate-of-return can be maximized. However, she cannot invest more than 5 million dollars and has to make sure that the overall investment cannot lose more than $25,000 in the worst-case scenario estimated based on the predicted rate-of-return.

(a) Formulate algebraically a linear programming model for the above problem. Define the decision variables, objective function, and constraints. (5 points)

(b) Draw the feasible region for the linear programming model. (2 points)

(c) Find the optimal solution(s) and optimal value of the objective function for the linear programing model. Justify why the solution is optimal. Describe also verbally how the manager should invest. (3 points)

(d) Suppose that the manager can also invest in bonds, which in the past had rate-of-return around 0.5%. The rate-of-return for bonds is known to be stable, so the manager predicts that even in the worst-case scenario the rate-of-return remains the same, i.e. 0.5%.
Formulate a linear programming model on a spreadsheet to find out how much the manager should invest in the energy sector, non-energy sectors, and bonds with the objective and requirements specified above. SOLVE using Excel solver (Provide a printout of the corresponding "Excel Spreadsheet" and the "Answer Report"). Describe verbally how the manager should invest. (5 points)

Problem 2 (6 points)

Consider the following linear programming model:
Minimize $x+0.5y$
Subject to

$$2y - x \le 1$$
$$y \le 1.5$$
$$x+2y \ge 2$$
$$2x - y \le 8$$
$$2x+3y \le 12$$
$$x \ge 1$$
$$x \ge 0, y \ge 0$$

(a) Draw the feasible region and objective function for the model. Show the optimal solution and optimal value of the objective function. Justify why the solution is optimal. (4 points)

(b) What other method can you choose to find the optimal solution without drawing the objective function? Considering the structure of the feasible region, which method is better? Justify your answer. (2 points)

Problem 3 (7 points)
Consider the following linear programming model:
Maximize $3x + y$
Subject to

$$2x + y \le 14$$
$$x - y \le 1$$
$$x + 12y \le 108$$
$$x \ge 3, y \ge 0$$

(a) Draw the feasible region and objective function for the model. Report what you find about the optimal solution(s) and the optimal value of the objective function. Justify your finding. (4 points)

(b) Is there any redundant constraint? Which one(s) and why? (1 point)

(c) Is there any constraint(s) that can be removed without changing the optimal solution(s) you obtained in (a) and why? (2 points)

Problem 4 (6 points)

Consider the following linear programming model:
Maximize $-2x + 0.5y$
Subject to

$$y - 4x \le 1$$
$$4x + y \ge 4$$
$$y \le 6$$
$$y \ge 0$$
$$x \ge 0.5$$

(a) Draw the feasible region and objective function for the model. Report what you find about the optimal solution(s) and the optimal value of the objective function. Justify your finding. (4 points)

(b) Change the objective function in the LP model to "2x-0.5y". Report what you find about the optimal solution(s) and the optimal value. Justify your finding. (2 points)

Problem 5 (6 points)

Consider the following linear programming model:
Maximize $2x - y$
Subject to

$$x+5y \le 12$$
$$x - 2y \le 6$$
$$x - y \ge 0$$
$$x+2y \le 8$$
$$x+3y \le 9$$
$$x \ge 0,\ y \ge 0$$

(a) Draw the feasible region for the model, however DO NOT draw the objective function. Without graphing the objective function (i.e. use the corner point method), find the optimal solution(s) and the optimal value of the objective function. Justify your method and why the solution(s) you obtain is (are) optimal. (4 points)

(b) Add the constraint "x+5y>=12" to the linear programming model. Is the optimal solution the same as the one in (a)? If yes, justify your answer by highlighting the new feasible region. If not, provide the new optimal solution also by highlighting the new feasible region. In both cases, justify why the solution is optimal. (2points)