Problem Set 1

2. A seller has a single indivisible unit of a good that is of interest to a buyer. The seller decides on a price p to set for the good. She posts this price in her storefront, and this announcement is observed by the buyer, who lives nearby. A single unit of this good is also being sold on Ebay at a price $q > 0$. After observing the seller's price, the buyer decides whether to buy the good held by the seller or the good sold on Ebay. Buying the good on Ebay also entails paying a shipping fee of $c \in (0, q)$. The buyer derives value $v > q + c$ from consuming either sellers' good, and the seller derives no benefit from retaining the good.

   (a) Formulate the interaction between the seller and the buyer as a game in strategic form.

   (b) Find a Nash equilibrium of this game in which the buyer does not visit the seller.

   (c) Find a Nash equilibrium of this game in which the buyer visits the seller.

   (d) Using your answer from (c), what can you say about the possible prices that the buyer pays in *any* Nash equilibrium in which the buyer visits the seller?

4. Consider a model of Bertrand oligopoly. That is, n firms compete for a fixed pool of customers by setting prices, with the output each firm puts on the market given its price determined by industry demand, which depends on the profile of prices $p = (p_1, ..., p_n)$. More precisely, given p, the demand for firm $i = 1, ..., n$ is given by

$$q_i(p_i, p_{-i}) = \begin{cases} \frac{\bar{q}}{|\{j=1,...,n; p_j = \min p\}|} & \text{if } i \in \{j = 1, ..., n; p_j = \min p\}, \\ = 0 & \text{otherwise,} \end{cases}$$

where $\bar{q} > 0$ (in words, all firms that set the lowest price share an inelastic industry demand $\bar{q}$). Firms have unit cost of production $c > 0$.

(a) Write down this model of oligopoly as a game in strategic form.

(b) Derive a symmetric pure strategy Nash equilibrium for this game (i.e., a Nash equilibrium in which all firms play the same pure strategy). Argue that no other symmetric pure strategy equilibrium exists.

(c) What happens to the symmetric equilibrium price as $n \to \infty$. Interpret your result.

(d) Suppose that $n > 2$. Derive a pure strategy Nash equilibrium of this game that is not symmetric.

8. An aggressor country (A) and a defending country (D) are involved in a nuclear confrontation. There are 2 locations in country D that can be targets for the missiles of country A. Country A has 2 missiles available to send against any of country D's locations, and country D has 1 anti-missile battery available to defend either location. The anti-missile battery can destroy all missiles that target the location that it defends. If a location is undefended, then 2 million of country D's citizens will die if a single missile strikes, and 3 million citizens will die if two missiles strike. Country A wants to maximise the number of deaths in country D, and country D wants to minimise them.

   (a) Describe this interaction between the two countries as a game in strategic form.

   (b) Show that this game does not have a Nash equilibrium in pure strategies.

   (c) Find all the Nash equilibria in mixed strategies for this game.