Written Homework 7, Due April 11

Written homework is to be handwritten and handed in at the beginning of discussion. In extenuating circumstances you can notify your Teaching Assistant via email before discussion, and homework can be submitted via alternative means the same day of discussion, under agreement with your Teaching Assistant. Otherwise the homework will be considered late. Show all your work, correct answers without supporting work will not receive full credit.

Question 1 *(3 points) When approximating the value of a definite integral, you don't necessarily know how much error there is between the approximated value and the true value of the integral. However, a handy rule of thumb is that the error is generally no larger than the difference between two successive approximations. In other words, if you are using the Trapezoid Rule and the difference between T_8 and T_9 is less than 0.25, then the error for T_9 is probably less than 0.25. Use this rule of thumb to approximate the value of the following integral to within an error tolerance of 1. Use the Trapezoid Rule starting with T_2, T_3, etc., and work your way up until the difference between two successive approximations is less than 1.*

$$\int_0^{60} \frac{1}{x + \cos\left(\frac{\pi}{2}x\right) + 1}\, dx$$

Show all your work. Write down all the fractions but do NOT add them by hand; you can use a calculator to add the fractions, rounding to the nearest hundredth.

Question 2 *(4 points)*

(a) (3 points) Find a power series representation for $\dfrac{\ln(1+2t^2)}{t}$ *starting with the Taylor Series for natural log:*

$$\ln(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + \frac{z^5}{5} - \frac{z^6}{6} + \cdots$$

Show all of your work!

(b) (1 point) Use the first two non-zero terms of your series in part (a) to estimate the integral:

$$\int_{\frac{1}{2}}^{1} \frac{\ln(1+2t^2)}{t}\, dt$$

Question 3 *(5 points) Let i be the imaginary number defined by the property $i^2 = -1$.*

(a) (1 point) Evaluate $i^3 =$ *and* $i^4 =$.

(b) (3 points) Find a power representation for the following functions, you may use the MacLaurin series for $e^x, \cos x,$ *and* $\sin x$.

$e^{ix} =$

$\cos(x) =$

$i\sin(x) =$

(c) (1 point) Using your answers in part (b), can you find a relationship between the three functions: $e^{ix}, \cos(x), i\sin(x)$*? This is called Euler's formula.*

Question 4 *(3 points) Find parametric equations to describe the straight-line path of a boat in the ocean. The boat's motor is propelling it 10 mph west (relative to the water) while at the same time the ocean waves are carrying the boat 3mph north. Assume the positive x-axis is pointing east, the positive y-axis is pointing north, and the boat's path goes through the point* $(5, -6)$.