Written Homework 10, Tuesday, April 30

Written homework is to be handwritten and handed in at the beginning of discussion. In extenuating circumstances you can notify your Teaching Assistant via email before discussion, and homework can be submitted via alternative means the same day of discussion, under agreement with your Teaching Assistant. Otherwise the homework will be considered late. Show all your work, correct answers without supporting work will not receive full credit.

Question 1 *(10 points) Consider the matrices:*

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 3 \\ -2 & -1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

(a) (3 points) Determine D so that $A + B = 2A - B + D$.

(b) ((3 points) Solve for $\mathbf{x}$ in the linear system given by $C \cdot \mathbf{x} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$.

(c) (4 points) Evaluate $(A \cdot B) \cdot \mathbf{x}$ and $(B \cdot A) \cdot \mathbf{x}$. Are these expressions the same? Show all your work!

Question 2 *(6 points) Consider the matrices:*

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad J = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Evaluate:

(a) $A \cdot I$

(b) $I \cdot A$

(c) $A \cdot J$

(d) $J \cdot A$

(e) $I \cdot \mathbf{x}$

(f) $J \cdot \mathbf{x}$

Question 3 *(4 points) Let*

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix} \quad B = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$$

Show all your work to find
(a) (1 point) the inverse (if it exists) of A,

(b) (1 point) the inverse (if it exists) of B,

(c) (2 points) (if it exists) $\left(B^{-1}\right)^{-1}$.

Question 4 *(8 points) Consider the system of equations*

$$2x + 4y = 1$$
$$2x + 2y = 3.$$

(a) (3 points) How can you write this system in matrix form $A\mathbf{x} = \mathbf{b}$*?*

(b) (2 points) What is the inverse of the matrix A*?*

(c) (3 points) Solve the linear system for $\begin{bmatrix} x \\ y \end{bmatrix}$ *by multiplying both sides on the left by* A^{-1}*.*

Question 5 *(3 points) Let*

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

and

$$B = \begin{bmatrix} -1 & 3 \\ 0 & 3 \end{bmatrix}$$

and

$$C = \begin{bmatrix} -1 & 2 \\ 2 & -4 \end{bmatrix}$$

Use the determinants to determine whether the matrices $A, B,$ *and* C *are invertible. In case they are invertible find the inverse matrix.*

Question 6 *(4 points) (Review Problem)*
Use a partial fraction decomposition to evaluate: $\int \frac{dx}{x^2(x+1)}$