

Let  $N$  points be located on a plane (or on a sphere). Then you can build a triangulation as follows: three points  $A, B, C$  form a triangle if and only if the circle, passing through these three points, does not contain other points (it is better to consider the sphere, so that there are no triples of points on straight lines).

And now we can consider 1-triangulation, 2-triangulation etc., which are defined as follows. Let's call a  $k$ -triangle a triangle  $ABC$ , such that inside it (or outside) is exactly  $k$  points of a given number.  $K$ -triangles form a "cell complex".

For example, if there is an edge  $AB$ , then two  $k$ -triangles can join it: above and below. Our "complex" consists of all vertices, all edges of  $k$ -triangles and the  $k$ -triangles themselves. In this case, edges and triangles, we understand abstract, no matter what they are drawn on the plane. If they contain common points, we do not pay attention to this.

So the question is: when will the two-dimensional manifold be obtained? In other words, you need to have exactly 2 triangles adjacent to each edge, and around each vertex the triangles formed a disk like petals.

Hint: it is worth experimenting with different sets of points, calculate the Euler characteristic, etc.