Exercise 1: Given the contour as sketched in Figure 1, where all points (x, y, z) on C satisfy $x^2 + y^2 = 16$ and $0 < z < 10\pi$

1. Find a parametrization. Take into account the arrow in the figure, indicating the direction this curve is traversed.

2. Find the length from $(4, 0, 0)$ to $(4, 0, 10\pi)$

3. Find $\int_C y \sin(z) ds$

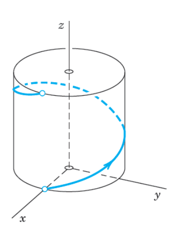

Figure 1: Circular Helix Line Path

Exercise 2: Consider the line integral:

$$\int_C (-y + \frac{1}{3}y^3 + x^2 y)ds \tag{1}$$

If C is a circle centred at the origin with radius r, for what values of r does the integral become 0?

Exercise 3: A hollow spherical bowl has an inner radius of R and a shell with thickness d, so the outer radius is $R+d$. See Figure 2. For the sake of reference,

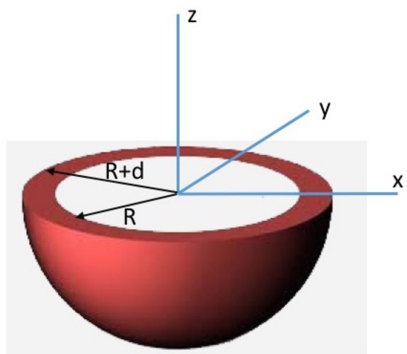

Figure 2: Hollow spherical bowl, corresponding to exercise 2.

we have indicated coordinates x, y, z. So the lowest point of the bowl is located at coordinates $x = 0$, $y = 0$, $z = -(R + d)$.

1. Give an expression for the volume occupied by the shell material in the form of a 3-d integral. Explicitly provide the bounds and the order of integration.

2. Compute this volume

3. The mass of the material of the shell has a density μ that is highest at the bottom of the shell of the bowl. This density is given by

   $$\mu(x, y, z) = -kz + 1,$$

   where k is a positive constant. Give an expression for the total mass of the shell in the form of a 3-d integral. Explicitly provide the bounds and the order of integration.

4. Compute the mass of the shell.

Hints: the volume of an object can be obtained by integrating 1 over this volume: $\int\int\int_V 1dV$, and the mass by $\int\int\int_V \mu dV$.

Exercise 4: Derive the Laplace equation in Spherical coordinates;

Use

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \tag{2}$$

To prove

$$\frac{\partial^2 u}{\partial r^2} + \frac{2}{r}\frac{\partial u}{\partial r} + \frac{cot(\phi)}{r^2}\frac{\partial u}{\partial \phi} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \phi^2} + \frac{1}{r^2 sin^2(\phi)}\frac{\partial^2 u}{\partial \theta^2} = 0 \tag{3}$$

Make sure to clearly state the coordinate transform you perform.

Exercise 5: Consider the sphere $x^2 + y^2 + z^2 = 2$ and the cone $z = \sqrt{x^2 + y^2}$

1. Find the surface area of the lower part of the sphere cut off by the cone.
2. Find the outward flux of $\mathbf{F} = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$ through this area.