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## 1. A Common Trig Integral from a few angles

### Gaining Familiarity

a. Compute  $\int_a^b 1 dx$ .

b. Compute  $\int_a^b \sin^2 x dx$  and  $\int_a^b \cos^2 x dx$  using the power-reducing formula.

c. Write a Pythagorean identity that relates these integrals.

i. Show that this equation is true using your computations from the integrals in parts a and b.

ii. Show that this equation is true with a geometric argument using graphs of these three functions

d. Find the smallest interval  $[a, b]$  so that  $p(x) = \sin x$  meets the first condition above.

e. Use the result from part c above to verify that  $\int_a^b \sin^2 x dx = \int_a^b \cos^2 x dx$  on the interval  $[a, b]$  you've identified.

### Using previous results

Recall the conditions from the last writing homework:

- $p(a) = p(b) = 0$
- $p''(x)$  exists for each  $x$  in  $[a, b]$ .

$$\int_a^b p(x)p''(x) dx = - \int_a^b (p'(x))^2 dx$$

that lead to the result

e. Use  $p(x) = \sin x$  along with the result from WH04 shown above to show

$$\int_a^b \sin^2 x dx = \int_a^b \cos^2 x dx$$

## 2. Area of a Circle, Again! (Ugh, Circles!? What is it with this guy and circles, huh!?)

Previously, we wrote functions that compute the area of the quarter circle in the first quadrant as a function of  $x$  and  $\theta$ .

We constructed the function A using calculus. From the left picture, we said that the area was

$$F(x) = \int_0^x \sqrt{1-t^2} dt$$

given by the function

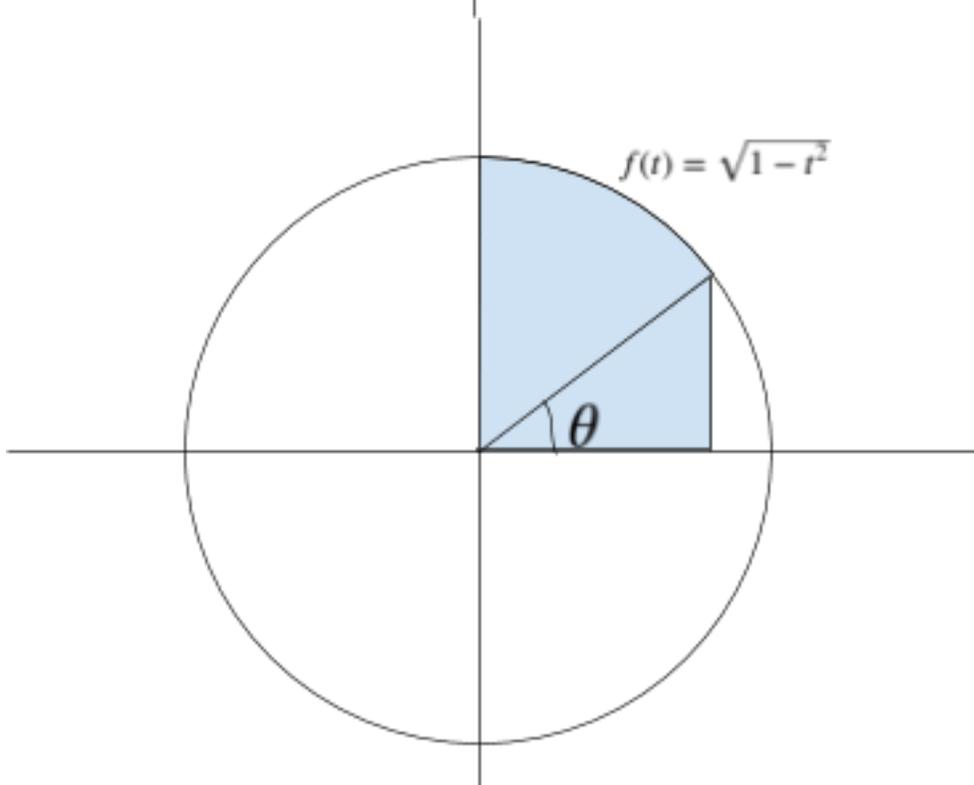
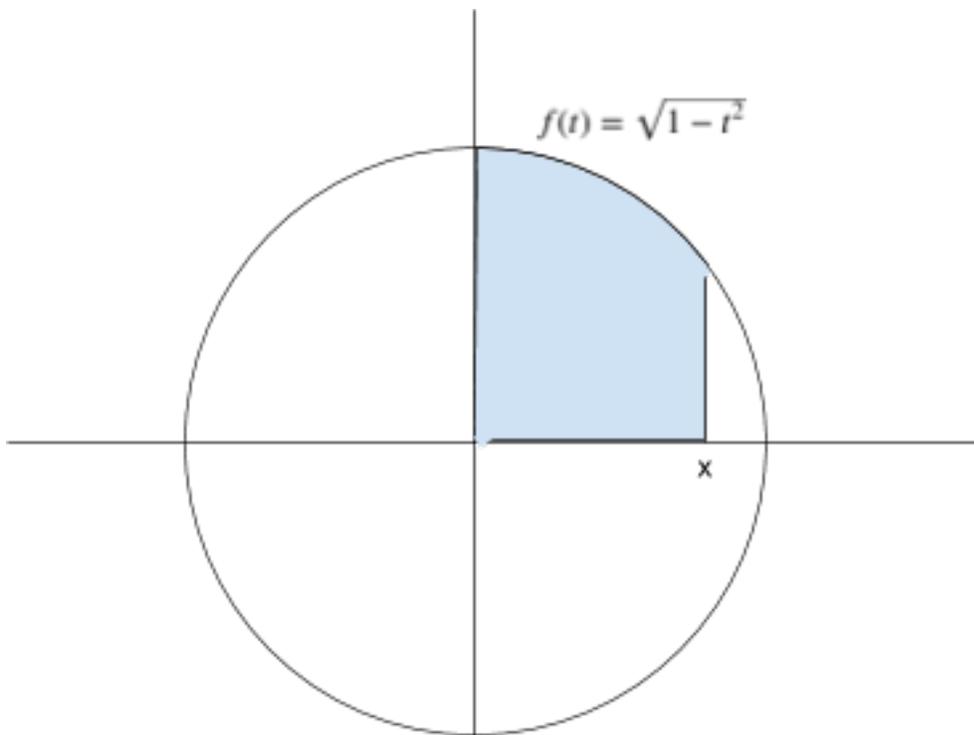
We constructed the function B using geometry. From the right picture, we said that the area was

$$G(\theta) = \frac{1}{2} \cos \theta \sin \theta + \frac{1}{2} \left( \frac{\pi}{2} - \theta \right)$$

given by the function

Next, we realized that since  $x = \cos \theta$ , then we can write

$$F(\theta) = \int_0^{\cos \theta} \sqrt{1-t^2} dt$$



$$\frac{d}{d\theta} F(\theta) = \frac{d}{d\theta} G(\theta)$$

On WHW02 we showed that  $\frac{d}{d\theta} F(\theta) = \frac{d}{d\theta} G(\theta)$ . This shows that F and G are the same function, except possibly for a constant.

a. For this writing homework, directly compute  $F(\theta)$ . I think the best way to do this is to

$$\int \sqrt{1-t^2} dt$$

compute an indefinite integral to find an antiderivative, then introduce the limits 0 to  $\cos(\theta)$ . You will use a trig substitution, but you'll need to use a variable other than  $\theta$ , since  $\theta$  is being used already.

b. Once you compute  $F(\theta)$ , show that this result is exactly  $G(\theta)$



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