1. A Common Trig Integral from a few angles

Gaining Familiarity

a. Compute $\int_a^b 1\,dx$.

b. Compute $\int_a^b \sin^2 x\,dx$ and $\int_a^b \cos^2 x\,dx$ using the power-reducing formula.

c. Write a Pythagorean identity that relates these integrals.

i. Show that this equation is true using your computations from the integrals in parts a and b.

ii. Show that this equation is true with a geometric argument using graphs of these three functions

d. Find the smallest interval [a, b] so that p(x) = sin x meets the first condition above.

e. Use the result from part c above to verify that $\int_a^b \sin^2 x\,dx = \int_a^b \cos^2 x\,dx$ on the interval [a, b] you've identified.

Using previous results

Recall the conditions from the last writing homework:

- p(a) = p(b) = 0
- p''(x) exists for each x in [a,b].

that lead to the result $\int_a^b p(x)p''(x)\,dx = -\int_a^b (p'(x))^2\,dx$.

e. Use p(x) = sin x along with the result from WH04 shown above to show

$\int_a^b \sin^2 x\,dx = \int_a^b \cos^2 x\,dx$.

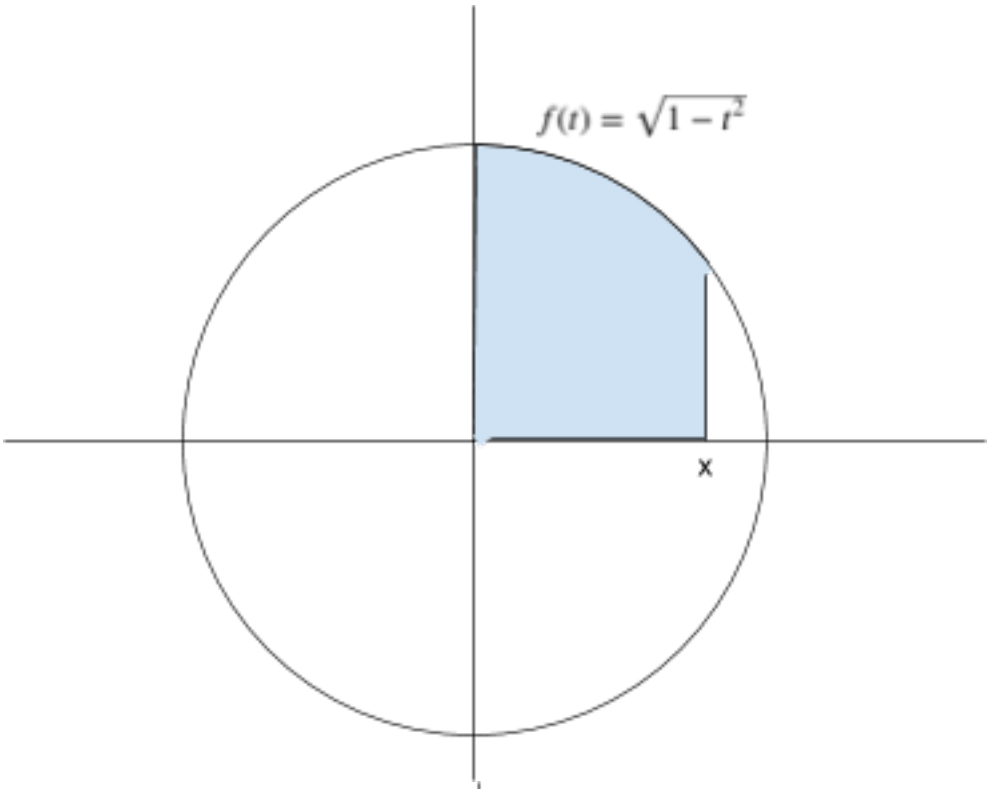
2. Area of a Circle, Again! (Ugh, Circles!? What is it with this guy and circles, huh!?)

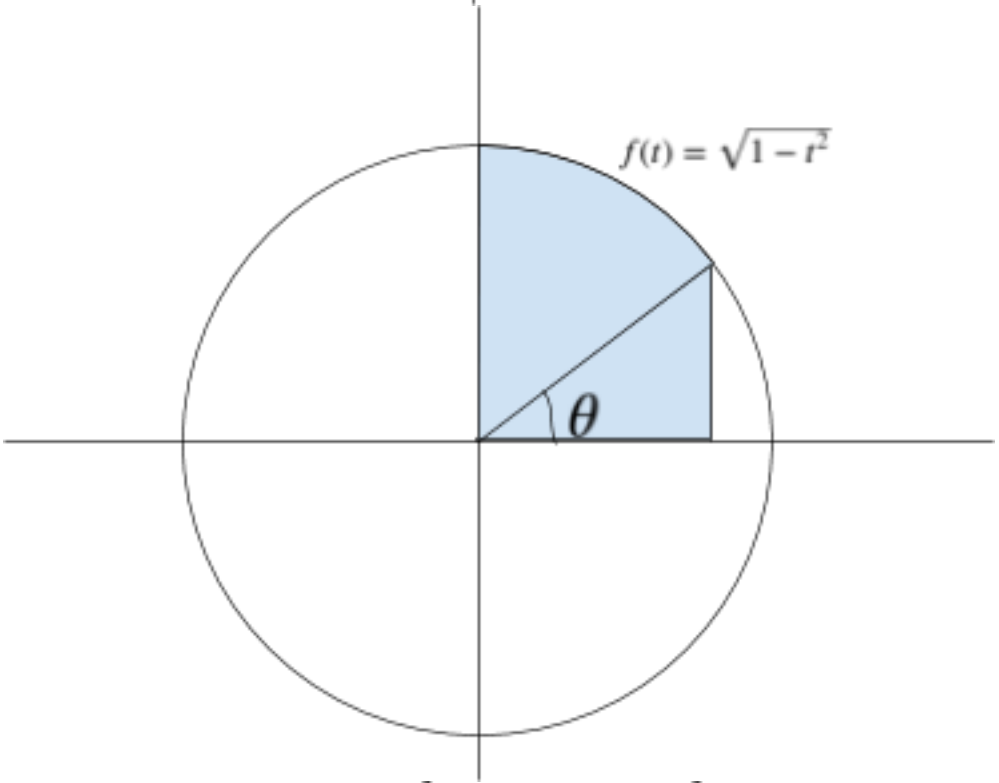
Previously, we wrote functions that compute the area of the quarter circle in the first quadrant as a function of x and theta.

We constructed the function A using calculus. From the left picture, we said that the area was given by the function $F(x) = \int_0^x \sqrt{1-t^2}\, dt$.

We constructed the function B using geometry. From the right picture, we said that the area was given by the function $G(\theta) = \frac{1}{2}\cos\theta \sin\theta + \frac{1}{2}\left(\frac{\pi}{2} - \theta\right)$.

Next, we realized that since $x = \cos\theta$, then we can write $F(\theta) = \int_0^{\cos\theta} \sqrt{1-t^2}\, dt$.





$$\frac{d}{d\theta}F(\theta) = \frac{d}{d\theta}G(\theta)$$

On WHW02 we showed that . This shows that F and G are the same function, except possibly for a constant.

a. For this writing homework, directly compute F(theta). I think the best way to do this is to compute an indefinite integral $\int \sqrt{1-t^2}\, dt$ to find an antiderivative, then introduce the limits 0 to cos(theta). You will use a trig substitution, but you'll need to use a variable other than theta, since theta is being used already.

b. Once you compute F(theta), show that this result is exactly G(theta)