1a Consider the following problem and partial solution. Fill in the details of this solution.

Let r be a constant real number. Compute the area described by the integral $\int_1^\infty \frac{1}{x^r}\,dx$.

As long as r is not equal to ????, we can compute as follows:

$$\begin{aligned}\int_1^\infty \frac{1}{x^r}\,dx = \int_1^\infty x^{-r}\,dx = \lim_{b\to\infty}\int_1^b x^{-r}\,dx &= \lim_{b\to\infty} \frac{1}{-r+1}x^{-r+1}\Big|_1^b \\ &= \lim_{b\to\infty} \frac{1}{-r+1}b^{-r+1} - \frac{1}{-r+1}1^{-r+1}\end{aligned}$$

If r does equal ????, we compute as follows:

Use your results above to describe the set of values of r for which this area is finite.

1b Compute the volume of the solid generated by revolving $y = \frac{1}{x^r}$ about the x axis for $x > 1$.

Describe the values of r for which this volume is finite.

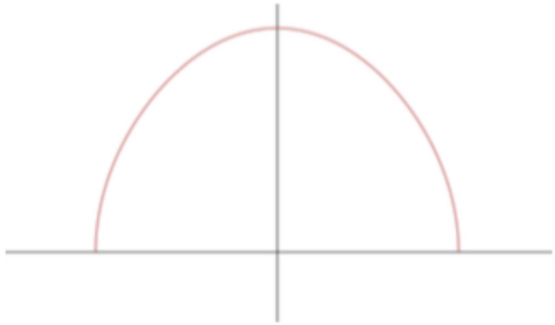
1c Think of the volume described in part b as a bucket full of water (with the positive x-axis representing the downward direction).Compute the work required to pump all of the water out of this infinite bucket.

Describe the values of r for which the work is finite.

1d Set up the integral for the surface area generated by revolving $y = \frac{1}{x^r}$ about the x axis for $x > 1$.

Describe the values of r for which the surface area is finite.

2 Below is a picture of part of the function $f(t) = \sqrt{\cos(2t)}$.



(a) Use this graph of f to sketch a graph of the function $F(x) = \int_0^x f(t)\, dt$ for all values of x defined by this part of f.

(b) Find the arc length of the curve $y = F(x)$. A trig identity may be useful for you once you set up and simplify your arc length integral.

3 You are building a water tank for a sprinkler system that drains from a hole in the bottom.

Your tank will be such that the volume of water left in the tank when the water level is h is described by the function $V(h) = \pi h^4$.

The rate at which the tank will drain (i.e. rate of change of volume with respect to time, t) is proportion to the square root of the height of the water.

(a) Write this draining condition as a differential equation involving V, t, and h. Describe what happens to the rate of change of volume as the height approaches zero.

(b) We know from the chain rule that $\frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt}$. Use this fact along with the volume function to rewrite your differential equation only in terms of h and t.

(c) Separate the variables and solve this differential equation. Arrive at an explicit description of the height h in terms of t. (Your answer will still involve the proportionality constant and the integration constant.)

(d) What happens to your function as t approaches infinity? Why does this make physical sense?