Problem Set 1

Due on June 26, 18pm, 2020

The goal of the problem set is to give you practice in mastering the course material. Try to give short, but precise answers, and always justify your solution.

You have to submit your solution individually but you are allowed to help each other when you cannot solve by yourself. When you do so, identify all your collaborators on the front page of your solution to avoid plagiarism charges.

If you obtain a solution through research (e.g., on Internet), you must acknowledge your source and write up the solution in your own words. It is a violation of this policy to submit a problem solution that you cannot orally explain to the professor.

Please type your answer using your favorite word processor or write in the provided spaces after printing this PDF file. You can submit your answers in MS Word or PDF file or scanned copy of your written answers. You can use additional pages if necessary.

You are expected to turn them in on Moodle by 18pm on June 26.

YOU
(NAME AND STUDENT ID NUMBER)

COLLABORATOR 1
(NAME AND STUDENT ID NUMBER)

COLLABORATOR 2
(NAME AND STUDENT ID NUMBER)

COLLABORATOR 3
(NAME AND STUDENT ID NUMBER)

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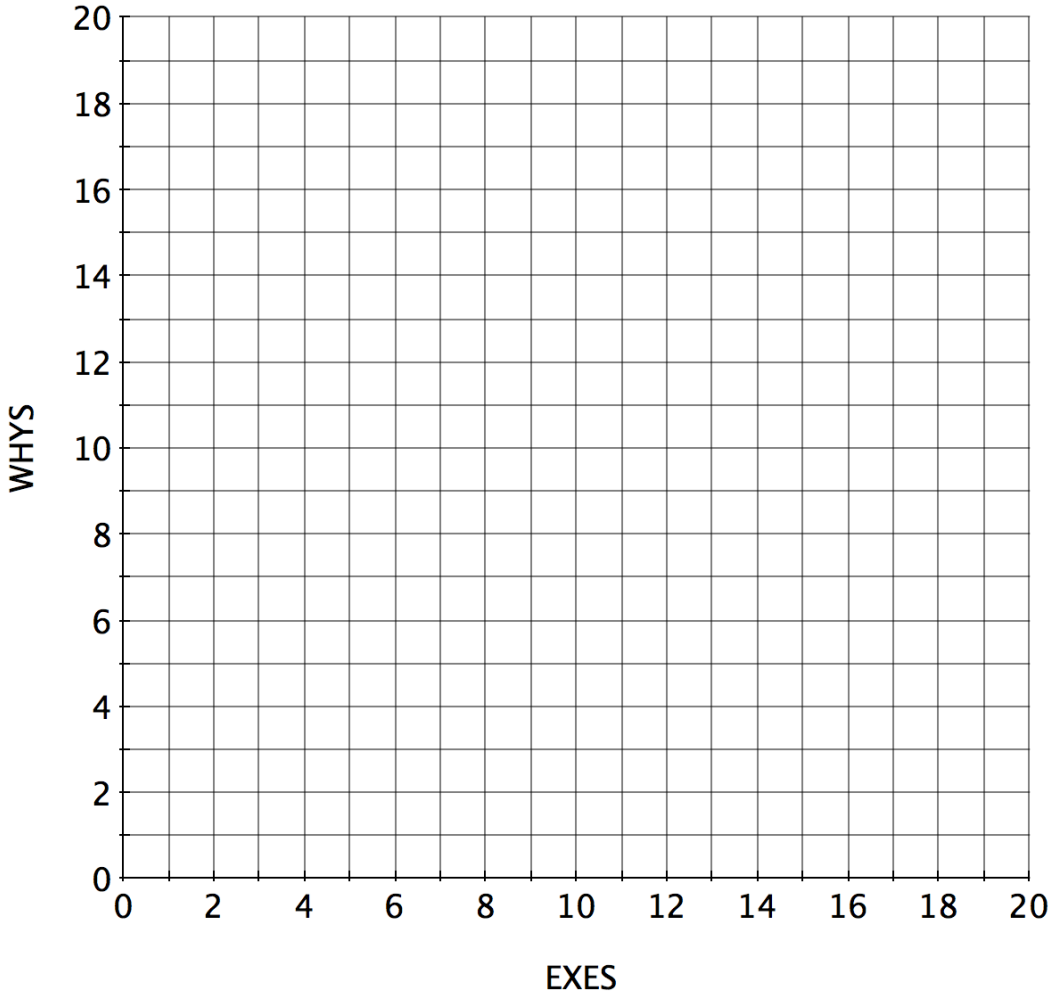
1. (40 points) Martha, John's neighbor, consumes only *exes* and *whys*, just like everyone else in the neighborhood. The consumption bundle where Martha consumes x units of *exes* per week and y units of *whys* per week is written as (x, y).[1]

   Martha has **quasilinear preferences** that are represented by the utility function $u(x, y) = 2\sqrt{x} + y$.

   (a) In the graph below, draw two of Martha's indifference curves: one that passes through the point $(1, 16)$ and an other that passes through $(4, 8)$.



4 points

[1] This exercise has been inspired by some problems that appear in the *Workouts in Intermediate Microeconomics* by Bergstrom and Varian. However, you will find less hints in this problem set than in the book. Should you need those hints, I suggest you to look at the original text (exercises 3.2, 4.2, 5.3, 6.3, and the appendix in chapter 6) .

Let us find Martha's demand function for *exes* and *whys*. Suppose that the unit price of *exes* is p_x, the unit price of *whys* is p_y and Martha's income is m.

(b) Compute and set the slope of Martha's indifference curve equal to the slope of the budget line, and solve for x.

4 points

(c) Use your previous result for x and plug it into the budget equation to find y.

2 point

(d) What is the minimum income level that makes sure that Martha consumes a non-negative amount of both goods?

4 point

(e) What is the optimal level of consumption of *exes* and *whys* if Martha's income is less than the minimum above? Why?

4 points

(f) Write (the mathematical expression for) Martha's demand function for *exes* and her demand function for *whys*.

5 points

(g) Use your answer to the previous question and complete the table below with Martha's optimal bundle for different price-income combinations.

p_x	p_y	m	x^*	y^*
2	4	40		
4	4	40		
2	4	20		
2	4	6		

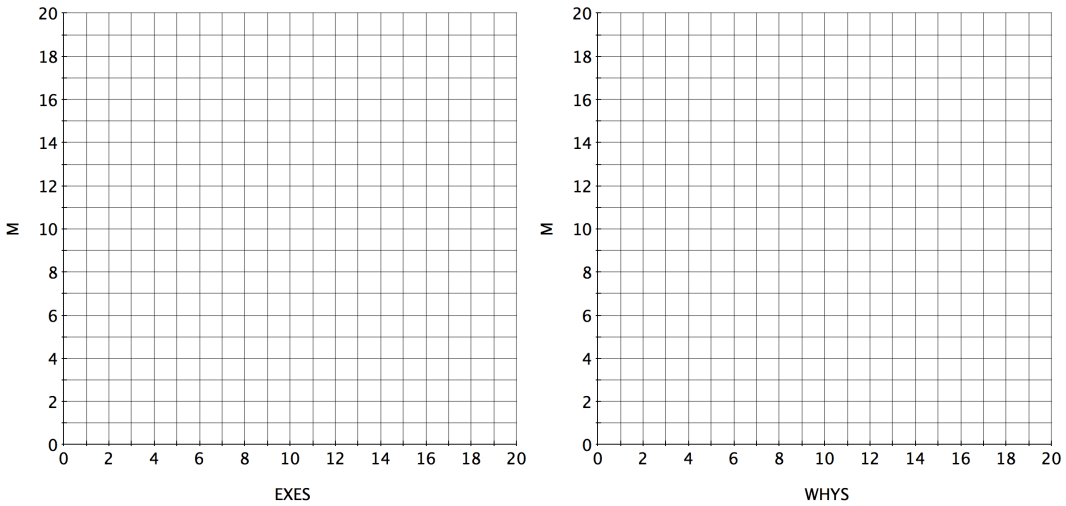
4 points

(h) Assume now that $p_x = 2$ and $p_y = 4$, and draw the following functions in the two graphs below.

i. Martha's Engel curve curve for *exes*.

ii. Martha's Engel curve curve for *whys*.

4 points



Assume again that the unit price of *exes* is 2, the unit price of *whys* is 4, and that Martha's income is 40. Now the local government levies a new tax on *exes* that increases the unit price of *exes* to 4.

(i) How does Martha's consumption of *exes* change as a consequence of the new tax?

4 point

(j) What part of the above change in demand is due to the substitution effect, and what part is due to the income effect?

5 points

2. (20 points) Katie, a consumer with a fixed income (m), only consumes burgers and pickled cucumbers (x_1 and x_2) and firmly believes that they should be consumed in fixed proportions: two burgers with three pickled cucumbers.

(a) Find the utility function that represent Katie's preferences. Are they homothetic? Explain.

5 points

(b) Write and solve the consumers utility-maximization problem to find her demand functions, $x_1(p_1, p_2, m)$ and $x_2(p_1, p_2, m)$.

5 points

(c) Using your previous results, try replacing the x_1 and x_2 in the Utility function by the demand functions $x_1(p_1, p_2, m)$ and $x_2(p_1, p_2, m)$. This creates a new "indirect utility function" that is a function of only prices and income, $v(p_1, p_2, m)$.

4 points

(d) The indirect utility function from above (part c) maps out the maximum achievable utility level given prices and levels. How does this maximum achievable utility level change if ...

6 point

i. ... income increases?

ii. ... prices increase?

iii. ... both income and prices double?

3. (26 points) Tom enjoys both yogurt (y) and xiaolongbao (x).[2] However, the grocery near his house keeps changing prices, as does his daily grocery budget.

Suppose Tom consumes the following bundles on different days when the prices are as follows:

Day	p_x	p_y	x	y
Day 1	5	2	3	2
Day 2	1	2	2	3
Day 3	4	2	5	2
Day 4	2	6	6	1

(a) Similar to the lecture slides, fill in the table below with the necessary budget for each bundle. In the table created, circle the bundles that were affordable on any given day.

2 points

	$x = 3, y = 2$			
($5,$2)	$19			

(b) Graph all 4 bundles and 4 budget lines on a graph. Label the axes and all 8 intercepts.

4 points

[2]Note: xiaolongbao are little dumplings typically filled with broth and meat.

(c) Are there any bundles that are directly revealed preferred to other bundles? If so, list out all directly revealed preferred bundles, and explain the general intuition for how the table in question A helps you determine directly revealed preferred bundles.

2 points

(d) Does Tom's choice behavior satisfy the Weak Axiom of Revealed Preference? If not, list all revealed preference relations that violate it. How many pairs are inconsistent (violations of WARP) ?

3 points

(e) Are there any bundles that are indirectly revealed preferred to other bundles? If so, list them all out, and explain the general intuition.

2 points

(f) Does Tom's choice behavior satisfy the Strong Axiom of Revealed Preference? If not, list all revealed preference relations that violate it. How many pairs are inconsistent with SARP but are not violations of WARP (that is, you do not need to relist violations of WARP from part d)?

3 points

(g) Can we create a utility function that describes Tom's preferences? If so, please do so. If not, why not?

3 points

(h) Suppose Tom's friend Sylvia also consumes different bundles on different days, as according to the following table. Does Sylvia's preferences adhere to WARP and SARP? If not, prove it by showing at least one violation. If so, demonstrate it with a table similar to part (a) and discuss why.

Day	p_x	p_y	x	y
Day 1	5	2	12	60
Day 2	1	2	60	60
Day 3	4	2	15	60
Day 4	2	6	30	20

3 points

(i) Are Sylvia's preferences best represented by a perfect substitutes utility function, a quasi-linear utility function, a Cobb-Douglas utility function, a perfect complements utility function, or are her preferences unable to be explained by a utility function? Please provide support and reasoning to justify your answer. (Hint: You can review the demand functions for each of these cases in the slides or text book. Using the demand functions as a guide may help determine the utility function being used.)

4 points

4. (14 points) Consider John who is considering purchasing red socks (R) and green socks (G). John, who has red-green colorblindness, considers them entirely interchangeable. That is, for John the socks are perfect substitutes.

(Note: Odd number of socks is fine as the laundry machine always eats some anyways. Fractional number of socks is also okay, they can be used to knit holes in other socks perhaps.)

(a) Initially the price of red socks is \$2 per sock, and the price of green socks is \$3 per sock. If John has \$60 to buy socks, how many red socks does he purchase? How many green socks does he purchase?

2 points

(b) During Lunar New Year, the price of red socks increases to \$5 per sock. Green socks remain \$3 per sock and his sock budget is still \$60. How many red socks does he purchase? How many green socks does he purchase?

2 points

(c) John decides he wants to take his purchasing behavior to the company and discuss how much the price changes affected him. If John calculates the Laspeyres index, how much would it indicate prices have risen by? If John calculates the Paasche index, how much would it indicate prices have risen by?

2 points

(d) Which of the two indices above is larger (representing a bigger price increase) ? Intuitively, why does that make sense?

2 points

(e) Jane is also a big fan of colored socks. However, unlike John, she believes every green sock should be worn along side a red sock, in the spirit of the holidays. That is, they are perfect complements for Jane. With the initial prices of \$2 per red sock and \$3 per green sock, and a budget of \$100 for socks, how many red socks does Jane buy? How many green socks does she buy?

2 points

(f) When the price increases for red socks to \$5, what happens to Jane's consumption – How many red socks does Jane buy? How many green socks does she buy?

2 points

(g) Jane is intrigued by John's decision to talk to the company about how much the price changes affected him. If Jane calculates the Laspeyres index for her consumption, how much would it indicate prices have risen by? If she calculates the Paasche index, how much would it indicate prices have risen by?

2 points

(h) How do Jane's price indices compare to John's? What's the intuition for this comparison?

2 points