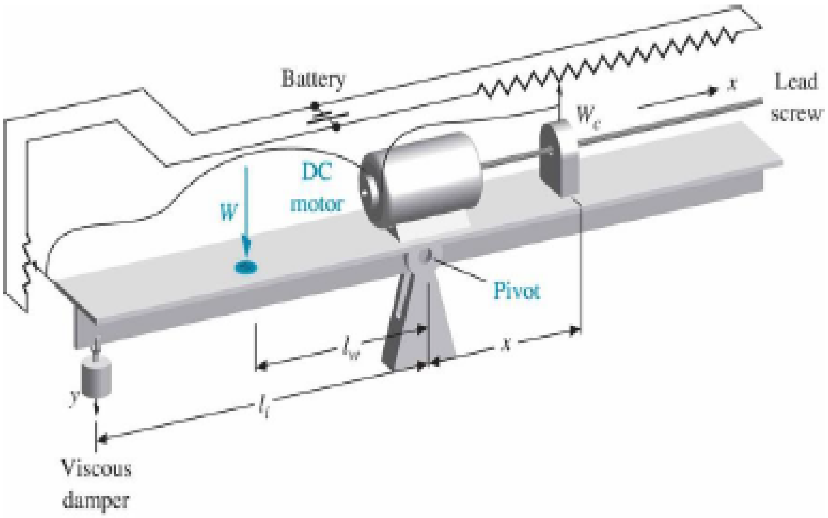
Problem 1: [50 points] Consider the automatic self-balancing scale system shown in the figure below



The parameters of the system are given as

Counterweight: $W_c = 2N$

Inertia of the beam: $I = 0.05kgm^2$

Length of the weight to the pivot: $l_w = 5cm$

Length of the beam to the viscous damper: $l_i = 20cm$

Damping constant of the viscous damper: $b = 10\sqrt{3}kg/m/s$

Lead screw gain: $K_s = \frac{1}{4000\pi} m/rad$

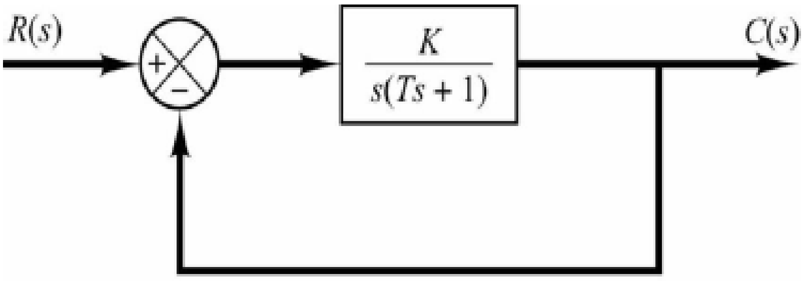
Input potentiometer gain: $K_i = 4800V/m$

Feedback potentiometer gain: $K_f = 400V/m$

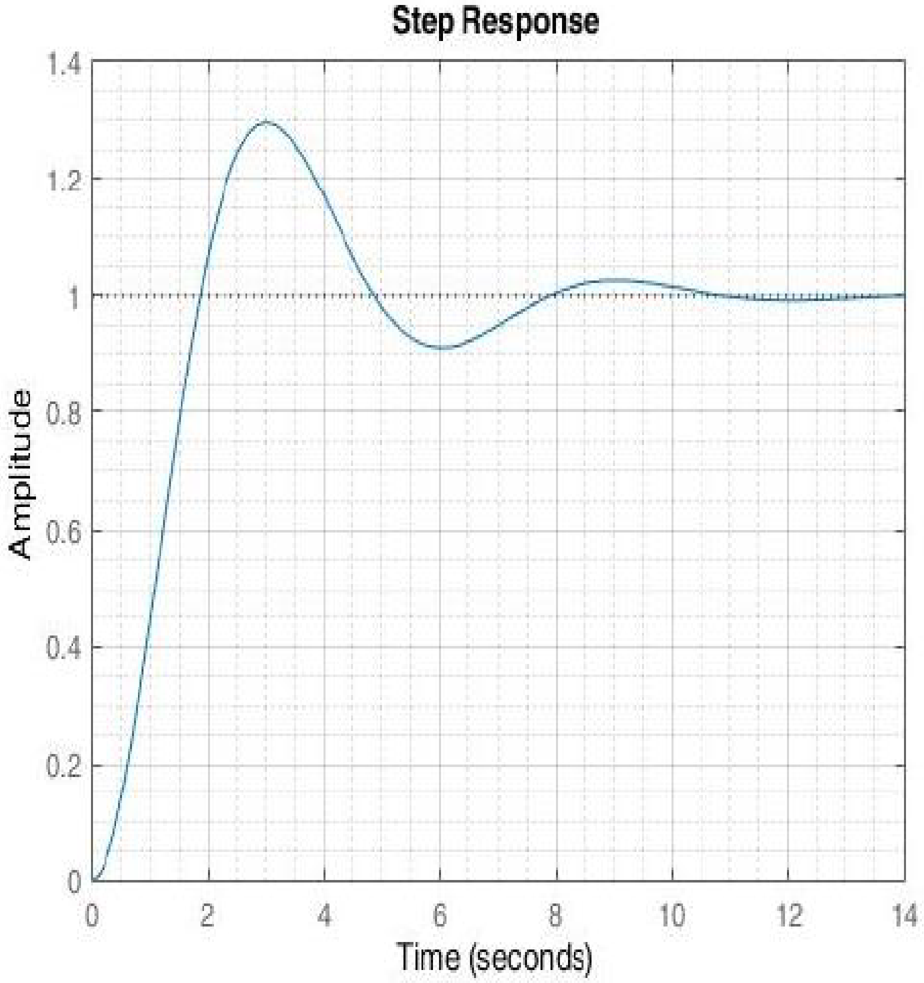
Assume that the transfer function of the DC motor is given by

$$G(s) = \frac{K}{s(Ts+1)}$$

where the parameters *K* and *T* are not known. Moreover, assume that in order to determine these parameters, the following unity feedback system has been constructed



The unit step response for this system is as shown below.



From the given step response determine the parameters *K* and *T* of the motor. For the steps below, do not perform simplification of the motor dynamics. In other words, utilize the motor transfer function given above with the parameters you obtained.

It is required to control the motor such that the system has at most 1 second settling time within 2% of the final value and at most 10% maximum overshoot. Also, the unit step response should have zero steady state error to the expected final value at the steady state (DC gain) of the closed loop system. (Note here that the steady state gain of the closed loop system is less than one and step response will settle at the steady state gain instead of one.) Design a PD-controller of the form

$$G_c(s) = K_c(s+2)$$

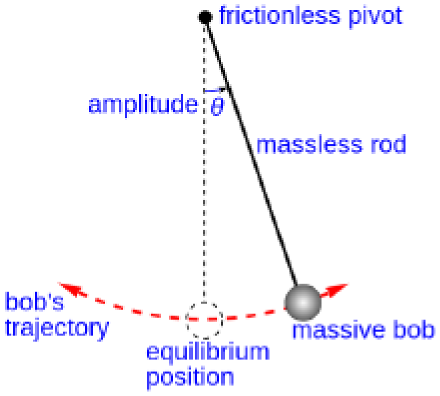
for the motor such that the above requirements for the overall system are satisfied. Sketch the overall block diagram of the system. Construct a mathematical model of the system components and the overall system and derive its transfer function. Simulate the response of the overall system and verify that the specifications are met. Show all your mathematical development, simulation code, and the obtained results.

Problem 2: [10 points] Consider the polynomial

$$p(s) = s^6 + 15s^5 + 9s^4 + 12s^3 + 20s^2 + 32s + 73$$

Using the Routh-Hurwitz stability criterion determine its roots which are located to the right of the vertical line passing through -1. (Perform the calculations manually. Show al your steps and reasoning.)

Problem 3: [40 points] Consider the pendulum shown below. It has two equilibrium points – one when the rod is oriented vertically down (first equilibrium point) and one when the rot is oriented vertically up (second equilibrium point).



Assume that the length of the rod is *L* and the mass of the bob is *M*. the angle of the pendulum with the vertical axis is denoted by θ and is shown in the figure. (The direction of the arrow denotes the positive direction for θ.) Assume that there is a motor at the pivot point which applies torque *T* to the pendulum. Assume that the pendulum is oriented up and operates close to the second equilibrium point.

a) Find the transfer function from the input *T* to the output θ of the system linearized about the second operating point.
b) Investigate the stability properties of the linearized system in part a).
c) Let *L* = 1m and *M* = 1kg. Take the gravitational acceleration as g = 9.8. Assume that there is device which measures the value of the angle θ. For the system in part a) design a controller which will result in a minimum phase critically damped closed loop system whose sinusoidal transfer function has the same corner frequencies as the sinusoidal transfer function of the open loop system. Verify your design.