Numerical Methods-Assignment-3
System of Linear Equations:Gaussian Elimination, Modified Gaussian Elimation with partial pivoting, Thomas algorithm and Gauss-Jordon Method

(1) Solve the following systems of linear equations using naive Gaussian elimination method, and modified Gaussian elimination method with partial pivoting and compare the component wise error involved in its solutions.

(i)

$$\begin{aligned} 0.729x_1+0.81x_2+0.9x_3 &= 0.6867, \\ x_1+x_2+x_3 &= 0.8338, \\ 1.331x_1+1.21x_2+1.1x_3 &= 1. \end{aligned}$$

(ii)

$$\begin{aligned} x_1-x_2+3x_3 &= 2, \\ 3x_1-3x_2+x_3 &= -1, \\ x_1+x_2 &= 3. \end{aligned}$$

(2) Count the number of operations involved in finding a solution using naive Gaussian elimination method to the following special class of linear systems having the form

$$\begin{aligned} a_{11}x_1+\ldots+a_{1n}x_n &= b_1, \\ &\ldots \\ &\ldots \\ a_{n1}x_1+\ldots+a_{nn}x_n &= b_n, \end{aligned}$$

where $a_{ij}=0$ wherever $i-j\geq 2$. In this exercise, we assume that the naive Gaussian elimination method has been implemented successfully. You must take into account the special nature of the given system.

(3) Solve the system

$$\begin{aligned} 0.001x_1+x_2 &= 1, \\ x_1+x_2 &= 2, \end{aligned}$$

i) using Gaussian elimination with partial pivoting with infinite precision,
ii) using naive Gaussian elimination with 2-digit rounding, and
iii) using modified Gaussian elimination method with partial pivoting, using 2- digit rounding.

(4) Use Thomas method to solve the tri-diagonal system of equations for $h=0.1$ and $h=0.01$,

$$\begin{aligned} -2(1+h^2)x_1+x_2 &= 1, \\ x_{i-1}-2(1+h^2)x_i+x_{i+1} &= 0, i=2,3, \\ x_3-2(1+h^2)x_4 &= 1. \end{aligned}$$

(5) Use Gauss-Jordon method to find the inverse of the matrix for the system of linear equations

$$\begin{aligned} x_1-\frac{1}{2}x_2+\frac{1}{3}x_3 &= 1, \\ \frac{1}{2}x_1-\frac{1}{3}x_2+\frac{1}{4}x_3 &= 2, \\ \frac{1}{3}x_1-\frac{1}{4}x_2+\frac{1}{5}x_3 &= 3, \end{aligned}$$

with (i) infinite precision and (ii) four-digit rounding.