Numerical Methods-Assignment-4
LU Decomposition, Doolittile, Crout and Cholesky methods

(1) Solve the following systems of linear equations using Doolittile's factorization method

$$\begin{aligned} 4x_1 + x_2 + x_3 &= 4, \\ x_1 + 4x_2 - 2x_3 &= 4, \\ 3x_1 + 2x_2 - 4x_3 &= 6. \end{aligned}$$

(2) Perform the Crout's decomposition method for the following system of equations

$$\begin{aligned} a_{11}x_1 + \ldots + a_{1n}x_n &= b_1, \\ &\ldots \\ &\ldots \\ a_{n1}x_1 + \ldots + a_{nn}x_n &= b_n, \end{aligned}$$

for $n = 5$ where $a_{ij} = 0$ wherever $i - j \leq 2$.

(3) Prove or disprove the following statements:
(i) An invertible matrix has at most one Doolittle factorization.
(ii) If a singular matrix has a Doolittle factorization, then the matrix has at least two Doolittle factorizations.

(4) Show that the matrix

$$\begin{pmatrix} 2 & 2 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 1 \end{pmatrix}$$

is invertible but has no LU factorization. Do a suitable interchange of rows to get an invertible matrix, which has an LU factorization. What can you conclude from your observation?

(5) Use Cholesky factorization to solve the system of linear equations

$$\begin{aligned} x_1 - 2x_2 + 2x_3 &= 4, \\ -2x_1 + 5x_2 - 3x_3 &= -7, \\ 2x_1 - 3x_2 + 6x_3 &= 10. \end{aligned}$$