HARRELL
PRECALCULUS

FUNCTIONS TEST
10.29.20
100 Points
DUE: November 3

Rules of the Road

- Complete this exam on your own paper or document and submit. You must show all work that leads to each answer you provide. Answers without work will not receive more than half credit. For problems that require significant work, your answers will receive minimal credit.
- All graphs are best done on graph paper, if you have access to it. If you complete graphs on notebook/unlined paper, they must be clear and reasonably accurate.
- To complete these problems, you may either work alone or with one other person (a partner). If you work with a partner, you will turn in one copy of the test and you will both receive the same grade for the test regardless of how you divide the work. If you are working alone, it is expected that you will work by yourself.
- If you are stuck, you should reach out to Mr. Harrell. You can use the internet for help, but be mindful not to plagiarize. Plagiarism is very easy to spot and will result in a zero for every problem in which it's evident.

Part I: Basic Skills.

1. Find the domain and range of the function $F(x) = -\sqrt{16 - x^2}$. Show the work that leads to your answer.

2. Graph the function $H(x) = 4 - |x + 1|$ on the coordinate plane. Next to your graph, indicate the domain and range of the function.

3. For the piecewise function $f(x) = \begin{cases} 3 - x, & x > 2 \\ |x^3|, & -2 \le x \le 2 \\ -\sqrt{-x}, & x < -2 \end{cases}$ evaluate $f(-2)$ and $(f \circ f)(7)$.

4. Consider the function $y = \left(\frac{3}{2-3x}\right)^3$. Find the functions $f(x)$, $g(x)$, and $h(x)$ so that $y = (f \circ g \circ h)(x)$

5. The function $f(x) = 3 - x^2$ does not have an inverse.
   a. Explain why *f* doesn't have an inverse. Use a graph to justify your answer.
   b. Create a partition of *f(x)* so that the new functions will have inverses. Find each inverse.

Part II: Fun Problems

6. Similar to the 'greatest integer function' (the 'floor' function), we can define the 'least integer function' ('ceiling' function). The least integer function maps any real number to the least integer greater than or equal to it: for positive numbers, it rounds numbers up. We denote the ceiling function as $L(x) = \lceil x \rceil$.

    a. Complete the following table:

x	14.01	2	½	-5.4	$-\pi$
$L(x) = \lceil x \rceil$					

    b. What is the domain and range of *L(x)*?

    c. Graph the least integer function for $-4 \leq x \leq 4$ on graph paper.

    d. In a few sentences, explain the differences and similarities between the floor and ceiling functions.

    e. Graph the function $K(x) = \lceil 2x \rceil$ for $-4 \leq x \leq 4$ on a separate set of axes.

7. A local fair charges a flat rate of $7.50 per person for admission to the fair grounds. In order to play games or buy food, the fair sells tickets. The first 50 tickets a customer buys cost $0.60 each. Starting with the 51st ticket, the price of a ticket drops to $0.50. The fair offers a roll of 100 tickets for a flat rate of $50. Individuals can only purchase up to 100 tickets at a time.
    a. Write a function $C(t)$ that models the cost (C) of attending the fair for *four people* and purchasing *t* tickets.
    b. Explain why this function must be a piecewise-defined function.
    c. Graph the function.
    d. Write the *real-world* domain and range of this function.
    e. Is the flat cost of 100 tickets a good deal? How much money will it save you? Explain how you know.
    f. You and your family of four estimate that you need 120 tickets to enjoy yourselves. Can you use this function to determine your cost? Explain your answer.

8. Carrie, a marine biologist, is performing experiments along the continental slope off of the coast of Baja California where the biodiversity is very dynamic. Scientists have proven that biodiversity is closely linked to the temperature of the water. Carrie uses a robot to monitor the temperature of the ocean at different depths along the continental slope, to help record the

changes in biodiversity due to changes in the temperature of the water. At the same time she does not want the robot to crash into the continental slope, so she needs to take into effects the speed of the currents.

Earlier scientists have found that the speed of ocean current as a function of depth. The speed of the ocean current, S, depends on depth, d, according to the following formula.

$$S(d) = 3d + 1$$

Where S is measured in meters per second and d is measured in meters. Suppose that the depth of a research robot depends on time, t, according to the formula:

$$d(t)=(1/27)t^2$$

1. Use function composition to write the speed of the current at the depth of the robot as a function of time. Give an exact expression.

2. What is the speed of the current at the depth of the robot after 9 seconds? Round your answer, if necessary, to the nearest integer.

3. What is the realistic domain of the robot and what does that represent?

4. What is the realistic range of the robot and what does that represent? (It might help to find the graph of S(t)).

9. Do this:
    a. IF YOU WORKED WITH A PARTNER: each of you needs to write a paragraph of the division of labor for the exam and what you found challenging and not-challenging in completing this test.
    b. IF YOU WORKED ALONE: write a paragraph on what you found challenging and not-challenging in completing this test.