Homework Information: The Problem Sets are based on material from class and various sources on Statistical Mechanics. There are five different categories of questions: computational $+$, computer c, concept ✓, derivation $\circ$ and math m. Show all work to receive full credit for the problem. Work together, ask questions, use whiteboards and have fun with it!

1. $+$ Consider a very large bag of marbles containing 1/3 red marbles, 1/3 blue marbles and 1/3 green marbles.

   (a) How many red, blue and green marbles are there in the predominant configuration if N marbles are drawn? From this, determine an expression W_{max}.

   (b) What is W for an arbitrary configuration of R red marbles, B blue marbles and G green marbles if N total marbles are drawn?

   (c) Use the results of part (a) and (b) to show that:

$$\ln \frac{W}{W_{max}} = R \ln \frac{N}{3R} + B \ln \frac{N}{3B} + G \ln \frac{N}{3G} \tag{1}$$

   (d) Let's say 3000 marbles were drawn. What is W/W_{max} when $R = G = 1$ and $B = 2998$? $R = G = 800$ and $B = 1400$? $R = B = G = 1000$? Is this what you would expect?

2. $+$ The barometric distribution can be used to model the density of particles as a function of height. Make sure your units stay consistent as you work through the problem.

   (a) Spherical particles of diameter 0.5 μm and density 1.10 g/mL are suspended in water (density 1 g/mL) at room temperature (293 K). What is the effective mass of one of these particles? This is the mass corrected for the fact that it is suspended in water rather than air. Calculate the vertical distance over which the number density of suspended particles decays by a factor of 2.

   (b) Calculate the vertical distance over which the density of oxygen molecules in the atmosphere decays by a factor of 2. Assume that the temperature is constant at 293 K over that distance, although that's a questionable assumption.

3. ✓ How would one go about using a measurement of particles suspended in water to experimentally determine the value of the Boltzmann constant?

4. $\circ$ Let's say we have an isolated macroscopic assembly of N harmonic oscillators. Now, if we focus on any three levels, $\epsilon_l < \epsilon_m < \epsilon_n$, a change in energy when an integer p units are moved from level m to level l and q units are moved from level m to level n. This allows us to write the expression:

$$\frac{\epsilon_n - \epsilon_m}{\epsilon_m - \epsilon_l} = \frac{p}{q} \tag{2}$$

(a) Using this, and the definition of W, along with assuming that the number of microstates is maximum at the original configuration, determine the following expression:

$$\frac{p}{q}\ln\frac{\eta_l}{\eta_m} = \ln\frac{\eta_m}{\eta_n} \quad (3)$$

Make sure to explicitly state all assumptions made in the derivation. Finally, substitute equation (2) into equation (3) and conclude the Boltzmann distribution law.

$$\frac{\eta_n}{\eta_0} = e^{-\beta\epsilon_n} \quad (4)$$

(b) What is β normally written as, in terms of only two variables? If you had two different systems, a cold temperature system and a hot temperature system, how would η_n/η_0 compare?

5. ° Recall that the number of microstates can be given by:

$$W = \frac{N!}{\Pi_i \eta_i!} \quad (5)$$

(a) Then show that when the total number of units is constant,

$$d\ln W = -\sum_i \ln\eta_i d\eta_i \quad (6)$$

(b) Use the fact that

$$\ln(\eta_i/\eta_0) = -\beta\epsilon_i \quad (7)$$

and the result of the previous problem to yield a final expression of

$$d\ln W = \beta dE \quad (8)$$