9.3 First-Order Linear Differential Equations

APPLICATION PREVIEW

Differential Equations and Drug Dynamics

When you swallow a pill, it dissolves in your stomach and the medication passes through your stomach lining into the plasma of your blood. Some of the medication is absorbed by the cells throughout your body, while the rest continues to circulate with your blood for later absorption. The amount $y(t)$ of medication remaining in the bloodstream t hours after swallowing the pill can be modeled by the differential equation

$$\frac{dy}{dt} = abe^{-bt} - cy$$

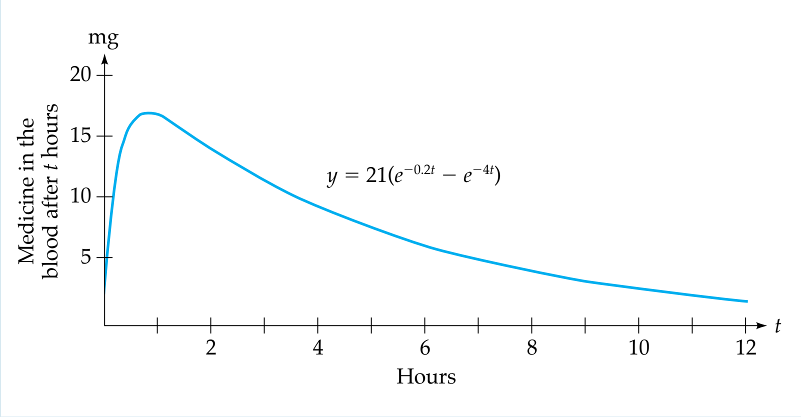
a = dosage of pill in milligrams (mg)
b = dissolution constant of pill
c = absorption constant of medicine

The first term on the right-hand side represents the rate at which the medicine enters your bloodstream (at first rapidly as most of the pill dissolves, and later more slowly when only a small amount of the pill remains), and the second term represents the rate at which the medication is absorbed from the blood.

This differential equation is not separable, but can be solved by the methods of this section. Its solution (assuming no medicine in the blood at time $t = 0$) is

$$y(t) = \frac{ab}{b - c}(e^{-ct} - e^{-bt})$$

This solution for the values $a = 20$ milligrams, $b = 4$, and $c = 0.2$ is shown below.



Notice that the amount of medicine in the bloodstream never reaches the pill dosage of 20 mg. This is because the medicine enters the bloodstream slowly, and some has already been absorbed before the pill has fully dissolved. By using the solutions of differential equations such as this, researchers can calculate dosages that are high enough to be effective yet low enough to avoid toxicity.