1. For the following relations, determine whether they are (1) reflexive (2) symmetric and/or (3) transitive. Give a justification or counterexample.

   (a) Let $x, y \in \mathbb{R}$. We define xRy if $x + y = 100$.

   (b) Let $x, y \in \mathbb{R}$. We define xRy if $|x - y| \leq 1$.

2. (a) The relation R on real numbers $\mathbb{R}$ is given by xRy if $x - y \in \mathbb{Q}$.
   (1) Show that R is an equivalence relation. (2) Describe all numbers in the equivalence class of 0, 1, and $\sqrt{2}$.

   (b) The relation R on pairs of real numbers $\mathbb{R} \times \mathbb{R}$ given by $(x, y)R(z, w)$ if $x + y = z + w$.
   (1) Show that R is an equivalence relation. (2) Describe all points in the equivalence class of $(4, 0)$ and the equivalence class of $(0, 0)$. Draw a sketch of the equivalence classes of this relation.

3. For the following relations, you do not need to show that they are equivalence relations. Just find the equivalence classes.

   (a) The relation R on the set of all twice differentiable functions is defined by fRg if $f''(x) = g''(x)$.
   Describe all functions that are in the equivalence class of $f(x) = x^3 + 2x$ and $g(x) = 0$. Hint: Be careful! Remember that every time you integrate a function, you need to add a constant....

   (b) Let R be the equivalence relation on the set $\mathbb{Q}$ of rational numbers defined by $\frac{a}{b} R \frac{c}{d}$ if $ad = bc$. Describe the elements of the equivalence class of $\frac{3}{2}$.

4. Consider the following partition of the set of integers $\mathbb{Z}$. Describe an equivalence relation whose equivalence classes are the elements of P.

$$P = \{\{0\}, \{1, -1\}, \{2, -2\}, \{3, -3\}, \{4, -4\}, \ldots\}$$

5. Describe the partition on $\mathbb{N}$ that arises for the following equivalence relation. For $m, n \in \mathbb{N}$, mRn if m and n have the exact same prime divisors.