Interpreting a Confidence Interval

We must be careful to interpret confidence intervals correctly. There is a correct interpretation and many different and creative incorrect interpretations of the confidence interval $0.405 < p < 0.455$.

Correct: **"We are 95% confident that the interval from 0.405 to 0.455 actually does contain the true value of the population proportion p."** This is a short and acceptable way of saying that if we were to select many different random samples of size 1487 (from Example 3) and construct the corresponding confidence intervals, 95% of them would contain the population proportion p. In this correct interpretation, the confidence level of 95% refers to the *success rate of the process* used to estimate the population proportion.

Wrong: **"There is a 95% chance that the true value of p will fall between 0.405 and 0.455."** This is wrong because p is a population parameter with a fixed value; it is not a random variable with values that vary.

Wrong: **"95% of sample proportions will fall between 0.405 and 0.455."** This is wrong because the values of 0.405 and 0.455 result from one sample; they are not parameters describing the behavior of all samples.

Confidence Level: The Process Success Rate A confidence level of 95% tells us that the *process* we are using should, in the long run, result in confidence interval limits that contain the true population proportion 95% of the time. Suppose that the true proportion of adults with Facebook pages is $p = 0.50$. See Figure 7-1, which shows that 19 out of 20 (or 95%) different confidence intervals contain the assumed value of $p = 0.50$. Figure 7-1 is trying to tell this story: With a 95% confidence level, we expect about 19 out of 20 confidence intervals (or 95%) to contain the true value of p.

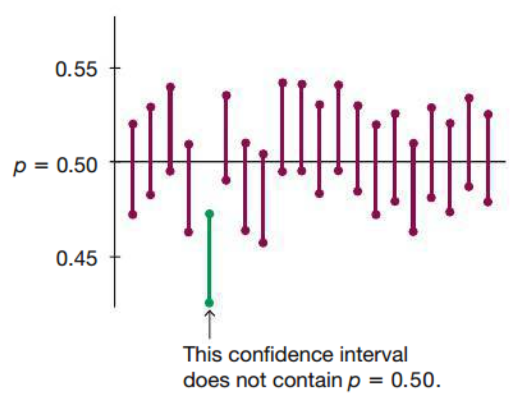

FIGURE 7-1 Confidence Intervals from 20 Different Samples

Critical Values

Critical values are formally defined on the next page and they are based on the following observations:

1. When certain requirements are met, the sampling distribution of sample proportions can be approximated by a normal distribution, as shown in Figure 7-2.
2. A z score associated with a sample proportion has a probability of $\alpha/2$ of falling in the right tail portion of Figure 7-2.
3. The z score at the boundary of the right-tail region is commonly denoted by $z_{\alpha/2}$ and is referred to as a *critical value* because it is on the borderline separating z scores that are significantly high.

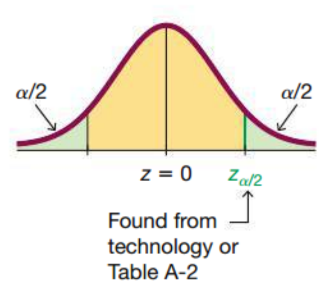

FIGURE 7-2 Critical Value $z_{\alpha/2}$ in the Standard Normal Distribution