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Stochastic Foundation of Finance (FIN 538)
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Problem Set 2

Problem 1 Consider the economy in Example 1, Lecture 1 which lasts for three quarters. Similar to Example 5 of Lecture 1, consider a security that pays $d_t = \$1$ if the economy state in quarter t is G and $d_t = \$0$ if the economy state in quarter t is B. Define

$$X_1 = d_1$$
$$X_2 = d_1 + d_2$$
$$X_3 = d_1 + d_2 + d_3$$

Note that both (d_1, d_2, d_3) and (X_1, X_2, X_3) are stochastic processes.

1. Suppose that the event $\omega = GBG$ is realized at the end of quarter 3. What are the values for d_1, d_2, d_3 and X_1, X_2, X_3?

2. Describe X_2 as a map from the sample space Ω (which you found in PS1) into the real line $\mathbb{R}$. Find the smallest sigma algebra that makes X_2 a random variable.

3. Suppose that d_1, d_2, d_3 are random maps on a common probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Find $\mathcal{F}$.

Problem 2 Let B_t be a Brownian motion.

1. Calculate $\mathbb{E}[B_t B_s]$ for $t \geq s$ where B_t .

2. Calculate $Var[B_t + B_s]$ for $t \geq s$ where B_t.

3. Calculate $\mathbb{E}[(B_s - B_t)^4], s > t > 0$.