

Problem Set 1

Problem (*An extension of Example 1 of Lecture 1*)

Suppose the economy in Example 1, Lecture 1 lasts for three quarters. Similar to Example 5 of Lecture 1, consider a security that pays $d_t = \$1$ if the economy state in quarter t is G and $d_t = \$0$ if the economy state in quarter t is B .

1. What is the sample space Ω ?
2. Following Example 1, find the filtration that corresponds to the σ -algebras \mathcal{F}_t at $t = 0, 1, 2, 3$.
(If the answer is too long, a short description in words will suffice)
3. Calculate the probability measure \mathbb{P} that is associated with each σ -algebra \mathcal{F}_t above for $t = 0, 1, 2, 3$.
(If the answer is too long, a short description in words will suffice)
4. Consider a security X with date-3 payoff defined as

$$X = d_1 + d_2 + d_3$$

Let Y be the payoff to a put option on X with a strike price of $K = 2$ and maturity of $T = 3$. Recall that the payoff for this call option is $Y = \max(K - X, 0)$.

- (a) Describe Y as a map: $Y : \Omega \rightarrow \mathbb{R}$.
- (b) Find the smallest possible σ -algebra that makes Y a random variable.