Homework 3

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3.6 The shelf life, in days, for bottles of a certain prescribed medicine is a random variable having the density function

$$f(x) = \begin{cases} \frac{20{,}000}{(x+100)^3}, & x > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the probability that a bottle of this medicine will have a shell life of

(a) at least 200 days;

(b) anywhere from 80 to 120 days.

3.7 The total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a period of one year is a continuous random variable X that has the density function

$$f(x) = \begin{cases} x, & 0 < x < 1, \\ 2 - x, & 1 \le x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the probability that over a period of one year, a family runs their vacuum cleaner

(a) less than 120 hours;

(b) between 50 and 100 hours.

3.11 A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets. If x is the number of defective sets purchased by the hotel, find the probability distribution of X. Express the results graphically as a probability histogram.

3.13 The probability distribution of X, the number of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width, is given by

x	0	1	2	3	4
$f(x)$	0.41	0.37	0.16	0.05	0.01

Construct the cumulative distribution function of X.

3.40 A fast-food restaurant operates both a drive-through facility and a walk-in facility. On a randomly selected day, let X and Y, respectively, be the proportions of the time that the drive-through and walk-in facilities are in use, and suppose that the joint density function of these random variables is

$$f(x, y) = \begin{cases} \frac{2}{3}(x + 2y), & 0 \le x \le 1,\ 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find the marginal density of X.

(b) Find the marginal density of Y.

(c) Find the probability that the drive-through facility is busy less than one-half of the time.

3.47 The amount of kerosene, in thousands of liters, in a tank at the beginning of any day is a random amount Y from which a random amount X is sold during that day. Suppose that the tank is not resupplied during the day so that $x \le y$, and assume that the joint density function of these variables is

$$f(x, y) = \begin{cases} 2, & 0 < x \le y < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Determine if X and Y are independent.

(b) Find P(1/4 < X < ½ | Y= 3/4).