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Stochastic Foundation of Finance (FIN 538)
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Problem Set 4

Problem In this problem we will see if the stochastic integral formula

$$\int_0^1 B_s(\omega)dB_s(\omega) = \frac{B_1^2}{2} - \frac{1}{2}$$

holds true in simulations. Use Excel to generate 100 sample paths of the Brownian motion in the time interval $[0, 1]$ by following the following steps:

1. Partition the interval $[0, 1]$ into 100 smaller intervals each of length 0.01: $t_0 = 0, t_1 = 0.01, t_2 = 0.01, \cdots, t_{100} = 1$.
2. For each t_i, generate a random number $B_{t_i} = B_{t_{i-1}} + \sqrt{t_i - t_{i-1}} * N(0, 1)$ where $N(0, 1)$ is the standard normal random variable.
3. Compute $I(\omega) = \frac{B_1(\omega)^2}{2} - \frac{1}{2}$ (note each ω corresponds to one simulation).
4. Compute the sum $S(\omega) = \sum_{i=1}^{100} B_{t_{i-1}}(\omega)(B_{t_i}(\omega) - B_{t_{i-1}}(\omega))$
5. Repeat the above process 100 times to get 100 sample paths of the Brownian motion. Compute
$$\bar{I} = \frac{\sum_\omega I(\omega)}{100}$$
and
$$\bar{S} = \frac{\sum_\omega S(\omega)}{100}$$
Are they close to 0? Why or why not?
6. Consider $\hat{S}(\omega) = \sum_{i=1}^{100} B_{t_i}(\omega)(B_{t_i}(\omega) - B_{t_{i-1}}(\omega))$ and define
$$\bar{\hat{S}} = \frac{\sum_\omega \hat{S}(\omega)}{100}$$
Is $\bar{\hat{S}}$ close to 0? Why or why not?

Please submit your code in the .xlsx together with your answers.