Math 2740 – Fall 2021
Assignment 5

Due **Thursday** 14 October at 10:00

General instructions/remarks

- Note that we are back to the regular schedule.
- This assignment consists of **two** parts:
  1. the *written* part consists of long form answers to mathematical problems;
  2. the *computer* part is answered using a `jupyter notebook`.
- Return the solutions to these problems as **two** files on UMLearn. See details in the individual parts. Two folders are available to submit your solutions, one for each assignment type.
- Late assignments will not be accepted and will be given a mark of **zero**. (The system is set to stop accepting submissions after the deadline and email submissions will be ignored.)
- The mark for Assignment 5 will be a percentage mark consisting of the average of the marks obtained for the written and computer parts.
- You must complete the assignment BY YOURSELF. Acts of academic dishonesty will be subject to academic discipline.

Written part

Instructions for the written part

- Return a single pdf file for the written part, indicating clearly on the first page your name, student number and the tutorial section you are registered in.
- Ensure the file you submit is legible, that all pages appear in the correct orientation, etc. Marks will be deducted for illegible submissions.
- Full marks are given to solutions that are not only correct but also are explained with sufficient amount of detail.
- It is possible that not all problems will be marked, but if you have not provided a solution for a problem that is marked, you will receive a **zero** for that question.

Problem 1. Find the singular value decomposition of the following matrices:

$$A = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} -2 & 0 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad C = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \end{pmatrix}.$$

Problem 2. Let $A \in \mathcal{M}_n$ be symmetric. Show that the singular values of A are the absolute values of the eigenvalues of A.

Problem 3. A symmetric matrix $A \in \mathcal{M}_n$ is **positive definite** if for all $x \in \mathbb{R}^n$, $x^T Ax > 0$. Show that if $A \in \mathcal{M}_n$ is positive definite, then its singular values equal its eigenvalues.

Problem 4. Compute the pseudoinverses of the matrices in Problem 1. [This is covered in the Lecture 09 part 2 video.]

Computer part

Instructions for the computer part

- Return a `jupyter notebook` (.ipynb) file. Files returned in other formats (PDF, R, etc.) will not be marked.
- Recall that the notebook **must** use `R`.
- The first cell in the notebook should be a `markdown` cell with your name, student number and tutorial section you are registered in indicated.
- Please make use of the capacity to insert `markdown` text in notebooks. In the same way as long form answers need some explanations to be worth full marks, I will need to be able to be told what you are doing in order for your code to receive full.

Create a `jupyter notebook` in which you verify your answers to the questions in Problems 1 and 4 in the *Written part* using `R` instead of computing by hand. Feel free to use existing `R` functions or work through the computations step by step, but make sure the results appear clearly in the same way as they would if you were working through the exercises by hand. (That is, if a function returns a list, do not just display the list but display explicitly the element in the list that answers the question at hand.)

For *bonus marks*, write your own functions to produce the required results, with the functions performing the different steps of the computation as you would do if you were computing by hand. (You can however use existing `R` functions for intermediate steps.)