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1. Numerical Integration and Differentiation. **50 Points**

When playing your favorite space simulator game, you launched a rocket from sea level. The problem is that your computer's graphics card drivers were corrupted and the only thing you could see was the velocity (m/s) and time (s) read outs and you recorded them in the table shown below:

Time	0	20	40	60	80	100	120	140	160	180	200	220	240	260
Velocity	0	139	298	433	685	1026	1279	1373	1490	1634	1800	1986	2191	2417

Time	280	300	320	340	360	380	400	420	440	460	480	500	520
Velocity	2651	2915	3303	3516	3860	4216	4630	5092	5612	6184	6760	7327	7581

Assume the ascent was completely vertical and following a straight line. We will use the above data to assess how our launch went.

Problem 1a. Numerical Integration:

Using Simpson's 1/3 rule and the Trapezoid rule, determine the rocket's approximate location at times t = 120 s, 240 s, 360 s, and 520 s. Recall that the integral of the velocity of an object with respect to time is equal to the position of the object. Use MATLAB to complete this problem.

Problem 1b. Numerical Differentiation:

Using the first order forward, backwards, and central differences methods generate a plot of the acceleration of the rocket over the entire launch considering a time step of 40s (note that the plot must have 3 curves, one for each differentiation method). Then, construct another similar plot but considering a time step of 20s. Use MATLAB to complete this problem.

2. **Solution of Ordinary Differential Equations.** **50 Points**

Estimate the solution of the following initial-value problem, on the interval 1 < t < 5.

Problem 2a.

Use the Euler's Method and the 4"-order Runge-Kutta method to approximate the solution to the ordinary differential equation, considering a time step of 0.1 seconds. Compare the numerical approximations and the analytical solution presented below by plotting them on the interval 1 < t < 5. Use MATLAB to complete this problem.

$$tx' + 2x = t^2 - t + 1 \qquad\qquad x(1) = 0.5$$

Problem 2b.

Calculate the true percentage error for the Euler's and 4"*-order Runge-Kutta methods at t = 2, 3, 4, and 5 seconds (a total of 8 true percentage errors). Use MATLAB to complete this problem.

$$x(t) = \frac{1}{4}t^2 - \frac{1}{3}t + \frac{1}{2} + \frac{1}{12t^2}$$