



**STUDYDADDY**

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1. For each of the statements below determine whether it is true or false, providing reasons for your answer. Let  $(X, \mathcal{T})$  be a topological space and  $A, B \subseteq X$  be two compact subsets.

(a) Is  $A \cup B$  compact? [2 marks]

(b) If  $(X, \mathcal{T})$  is Hausdorff, is  $A \cap B$  compact? [2 marks]

2. Let  $f : [a, b] \rightarrow \mathbb{R}$  be continuous.

(a) Prove that the map  $g : [a, b] \rightarrow \mathbb{R}^2, x \mapsto (x, f(x))$  is also continuous. [3 marks]

(b) Prove that the graph of  $f$  is a compact subset of  $\mathbb{R}^2$ . [3 marks]

3. Let  $(X, \mathcal{T})$  and  $(Y, \mathcal{S})$  be connected topological spaces. Show that  $X \times Y$  is connected in the product topology. [8 marks]

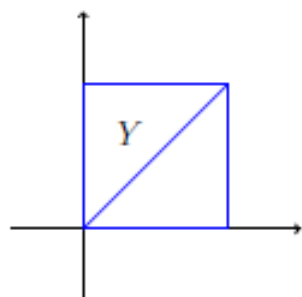
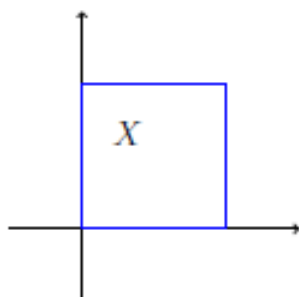
4. Consider the subsets [5 marks]

$$X = \{(x, y) : 0 \leq x \leq 1, y \in \{0, 1\}\} \cup \{(x, y) : 0 \leq y \leq 1, x \in \{0, 1\}\}$$

and

$$Y = X \cup \{(x, x) : 0 \leq x \leq 1\}$$

of  $\mathbb{R}^2$  with the standard topology from  $\mathbb{R}^2$ . Show that  $X$  and  $Y$  are not homeomorphic.



5. Consider the subset [8 marks]

$$Z = \{(0, 0), (0, 1)\} \cup \bigcup_{n=1}^{\infty} L_n$$

of  $\mathbb{R}^2$  (with the standard topology from  $\mathbb{R}^2$ ), where

$$L_n = \{(1/n, y) : 0 \leq y \leq 1\}$$

for all  $n \geq 1$ . Let  $U$  be a non-empty subset of  $Z$  which is both open and closed in  $Z$ . Show that if  $U$  contains one of the points  $(0, 0)$  and  $(0, 1)$ , then it contains the other as well.

6. Let  $\mathbb{RP}^2$  be the real projective space as defined in lectures, and let  $\pi : S^2 \rightarrow \mathbb{RP}^2$  be the natural projection. Consider  $\mathbb{R}^3$  and  $\mathbb{R}^4$  with the standard topologies, and let the function  $f : \mathbb{R}^3 \rightarrow \mathbb{R}^4$  be defined by

$$f(x, y, z) = (x^2 - z^2, xy, yz, xz).$$

(a) Show that there exists a function  $g : \mathbb{RP}^2 \rightarrow \mathbb{R}^4$  such that  $f = g \circ \pi$  on  $S^2$ . [4 marks]

(b) Show that  $g$  is continuous and one-to-one. [11 marks]

(c) Show that  $g$  is a homeomorphism between  $\mathbb{RP}^2$  and the subset  $Y = g(\mathbb{RP}^2)$  of  $\mathbb{R}^4$ . [4 marks]



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