PHIL 1300: Introduction to Logic

Problem Set 2

This problem set tests your understanding of the material covered in modules three and four. It deals with syntax, grammar, and symbolization in SL, and with the semantic concept of validity and with semantic properties of sentences in SL (truth tables). Be sure to attempt all of the practice exercises for modules three and four before submitting this problem set to your instructor.

1 Symbolization

Using the symbolization scheme given below, translate each of the following sentences into the language of sentential logic:

1. Vancouver is a capital city, but so is Regina.
2. If Ottawa is not a capital city then neither is Winnipeg.
3. Toronto is a capital city if Ottawa isn't.
4. Saskatoon is not a capital city only if Regina is.
5. At least one of Toronto, Montreal, and Winnipeg is a capital city.
6. No more than one of Toronto, Montreal, and Winnipeg is a capital city.
7. Saskatoon is not a capital city only if Regina is, in which case, Regina is a capital city.

Scheme:

- V: Vancouver is a capital city.
- R: Regina is a capital city.
- O: Ottawa is a capital city.
- W: Winnipeg is a capital city.
- T: Toronto is a capital city.
- S: Saskatoon is a capital city.
- M: Montreal is a capital city.

2 Sentences vs. Non-sentences

Which of the following strings of symbols count as (well-formed, or grammatical) *sentences* of SL?

- $(P \vee Q) \leftrightarrow \neg(R \wedge T)$
- $\neg\neg\neg P$
- $\varphi \wedge \psi$
- $\varphi \rightarrow (P \vee Q)$
- $P \wedge \neg Q$
- $P \wedge \neg \rightarrow Q$
- $P \rightarrow Q \rightarrow R$
- $[(P \leftrightarrow R) \wedge (R \leftrightarrow Q) \wedge P] \rightarrow Q$

3 Validity and Invalidity

Construct a complete truth table in order to determine whether the following arguments are valid. Be sure to indicate how you have arrived at your answer.

1. $P \leftrightarrow Q$; $Q \rightarrow (Q \vee R)$; $\neg(Q \vee R) \therefore \neg P$
2. $\neg T \vee (S \rightarrow R)$; $\neg(\neg T \vee R) \therefore \neg(\neg S \vee \neg T)$

4 Semantic Properties

For each of the following sentences, construct a truth table to determine whether the sentence is a tautology, a contradiction, or a contingent sentence. You may use a partial truth table if you wish.

1. $[\neg Q \leftrightarrow (P \rightarrow R)] \rightarrow (R \rightarrow Q)$
2. $\neg[P \rightarrow (Q \vee R)] \leftrightarrow [P \wedge (\neg Q \wedge \neg R)]$
3. $[(Z \wedge T) \leftrightarrow (P \wedge Q)] \wedge (\neg P \vee \neg Q)$