PHIL 1300: Introduction to Logic

Module Two

Contents

1 Overview

This second set of course notes covers additional material discussed in Chapter 2 of *Forall x: An Introduction to Formal Logic*, as well as the key logical concepts introduced in Chapter 3. Chapter 3 is the assigned reading for the second module of the course (at the point you should already have read Chapter 2 in its entirety). You should proceed by first carefully reading the assigned chapters, and then turn to these course notes. Once you have completed the reading, turn to the practice exercises assigned for this module.

This module of the course also contains your first **problem set**. Once you have completed the reading and practice exercises, you should complete and submit your problem set for grading.

2 Review

The first module of the course introduced the concepts of *logical consequence* and *argument validity*. This section briefly reviews those concepts and elaborates further on the concept of validity. Here are the key definitions once again:

Logical Consequence.[a] $B_1, B_2, \ldots, B_n \vdash A$ if and only if it is impossible that $B_1, B_2, \ldots, B_n$ are true and A is false.

[a] Recall that we are employing '$\vdash$' to represent the relation of logical consequence.

An argument from a set of premises $B_1, B_2, \ldots, B_n$ to a conclusion A is a **valid argument** if and only if $B_1, B_2, \ldots, B_n \vdash A$.

An argument from a set of premises $B_1, B_2, \ldots, B_n$ to a conclusion A is an **invalid argument** if and only if $B_1, B_2, \ldots, B_n \nvdash A$.[a]

[a] Recall that we are employing '$\nvdash$' to represent the failure of a sentence A to be a logical consequence of some set of sentences $B_1, B_2, \ldots, B_n$.

We know that an argument is valid whenever its conclusion is a logical consequence of its premises, taken together. At this point, you should be comfortable diagnosing whether simple arguments in English are valid. However, as an aid to your understanding of the nature of argument validity (and hence of logical consequence), it can be helpful to observe that arguments with the following logical properties count as valid arguments, by definition.

1. Arguments with *inconsistent premises*.
   - Consider this argument:
     1. Ange is taller than Britt.
     2. Britt is taller than Ange.
     3. ∴ The Earth orbits the moon.
   - That argument is technically valid. Why?
   - Well, consider the premises. Notice that it is impossible that both premises are true (if premise 1 is true, premise 2 is false, and vice-versa). This means that the premises are *inconsistent*.
   - But if the premises are inconsistent, then it is impossible for them to be (jointly) true, in which case it is impossible for the premises to be true and the conclusion false. Thus the argument is valid, by definition. In fact, *any* argument with inconsistent premises is guaranteed to be valid, for this reason.

2. Arguments with conclusions that are *necessarily true*.
   - Consider this argument:
     1. Ange is taller than Britt.
     2. Britt is older than Ange.
     3. ∴ $2 + 3 = 5$.

- The conclusion of this argument is a necessary truth (it could not be false).
- But if the conclusion of the argument could not be false, then it is impossible for the premises to be true *and* for the conclusion to be false, in which case the argument is (again) valid by definition.
- More generally, any argument with a necessarily true conclusion is guaranteed to be valid.

3. Arguments with *impossible premises*.

- Consider this argument:

  1. Paul is a married bachelor.
  2. ∴ The sun orbits the Earth.

- This argument is a bit like the first argument considered above. Its premise could not be true (it is *impossible* for a bachelor to be a married man).
- If so, it is impossible for the premise of this argument to be true, and for the conclusion to be false. Thus the argument is valid.
- More generally, any argument with impossible premises is guaranteed to be valid, for this very reason.

What we have observed in this section is that many arguments we would intuitively consider to be *bad* or *defective* count as valid. But the important thing to note at this stage is that their defective character does not derive from their invalidity. Many intuitively defective aruguments count as valid in the formal sense of validity at issue in this course. This should indicate to you that when we're considering whether an argument is compelling or persuasive, we're typically looking for more than just validity. What else are we looking for? That is the question taken up in the following section.

3 Soundness

3.1 What it is

In many cases, validity is 'easy to spot'. Some arguments are manifestly valid (or can be seen to be so on just a moment's reflection).

- Consider this argument:

  1. If you're human, then you're mortal.
  2. You're human.
  3. ∴ You're mortal.

- There is a fairly obvious sense in which the conclusion of that argument follows from its premises. The argument is plainly valid.
- Moreover, notice that in the above example all of the premises happen to be (actually) *true*.
- Whenever an argument has these features—that is, whenever an argument is *both* valid *and* has true premises—we say that the argument is *sound*. Here is the definition:

> An argument from a set of premises $B_1, B_2, \ldots, B_n$ to a conclusion A is a **sound argument** if and only if: (1) $B_1, B_2, \ldots, B_n \vdash A$ and (2) each of $B_1, B_2, \ldots, B_n$ is *true*.
>
> An argument from a set of premises $B_1, B_2, \ldots, B_n$ to a conclusion A is an **unsound argument** if and only if either (1) $B_1, B_2, \ldots, B_n \nvdash A$ or (2) at least one of $B_1, B_2, \ldots, B_n$ is *false*.

- Notice that each of the defective arguments we considered above in §2 fail this test. They all have at least one false premise. So despite the fact that they are all valid, they are each unsound.
- But another way for an argument to be unsound is for the argument to be invalid. That means there are three different ways for an argument to be unsound. An argument is unsound whenever:
  - It is invalid.
  - It has at least one false premise.
  - It is both invalid and has at least one false premise.

3.2 Why we care about it

Naturally, what we're interested in most of the time as reasoners is whether the arguments we are considering are *sound* arguments.

- That is because we (typically) want to form true beliefs about the world around us, and a sound argument is guaranteed to have a true conclusion. Whenever the premises in a valid argument are true (and hence, whenever the argument is sound), accepting or believing the conclusion of that argument is a sure-fire way of coming to form a true belief (the conclusion of a sound argument *must* be true).
- However, despite the fact that it is soundness that we typically care about when reasoning about the world around us, validity is an important property in its own right. And (setting aside simple examples like the ones we have been considering so far) it can in some cases be *very difficult* to determine whether a given argument has that property. Consider this argument, for example:

  1. Ange will be at the party only if Britt stays home.
  2. Britt never stays home.
  3. If Ange isn't at the party, then Cam will be despondent.
  4. Parties are never fun when Cam is desponent.

5. $\therefore$ The party will not be fun.

- As it happens, that is a valid argument.
- But it is much more difficult to see this at a glance than it is in the case of simple arguments like the one we discussed in Module One. Very soon, we'll be developing a formal system in which you'll be able to *demonstrate* to yourself that arguments like the above are valid.

4 Deductive and Non-deductive arguments

Some fairly good arguments are not sound, precisely because they are not valid. Suppose you were trying to convince me that the sun will rise tomorrow. Since you can't see into the future any better than I can, how would you do it? Well, one thing you might do is present me with an argument like this one:

1. Every single day since the beginning of the world, the sun has risen.
2. We have no special reason to think that tomorrow will be any different from every single day in the past.
3. $\therefore$ The sun will rise tomorrow.

- That's not a bad argument: if I was unsure about whether the sun will rise tomorrow (perhaps because I've been living in a cave my whole life), I'd probably be pretty convinced.
- But notice that the argument itself is *invalid*. Its premises (taken together) do not logically imply its conclusion, since it is *possible* that both of those premises be true, together, and yet false that the sun rises tomorrow.
- To see this, all you need to do is reflect on the fact that it's possible (for all we know) that the world ends at precisely midnight tonight, and so (in the possible circumstance we are considering) the sun doesn't rise tomorrow. (Maybe there will be no tomorrow!!)
- Argument like the above are called *non-deductive* arguments. Here is a definition:

> An argument from a set of premises $B_1, B_2, \ldots, B_n$ to a conclusion A is a **non-deductive** argument if and only if (1) $B_1, B_2, \ldots, B_n \nvDash A$ and (2) the truth of $B_1, B_2, \ldots, B_n$ nevertheless supports A.

Our focus at this stage of the course is with deductive arguments. Over the next several weeks, we will be studying the nature of deductive validity and logical consequence in detail, by developing a formal system in which those properties can be more precisely represented and understood. The final 1/3 of the course will deal with topics in the logic of non-deductive reasoning.

5 Other Logical Concepts

This section reviews some additional logical concepts that are important for an understanding of the core notions of logical consequence, deductive validity, and soundness.

5.1 Properties of sentences

5.1.1 Necessary truths and necessary falsehoods

Not all sentences are created equal! Some sentences are true, and others are false. But now consider the true sentences. Some of these—the *necessary truths*—are sentences that could not be false. Likewise, among the sentences that are false, some are such that they are *necessarily false*, and thus could not be true. Here are some definitions:

> **Necessary Truth**. Sentence S is a necessary truth if and only if it is impossible that S is false.
>
> **Necessary falsehood**. Sentence S is a necessary falsehood if and only if it is impossible that S is true.

Although not all philosophers and logicians agree on these matters, relatively uncontroversial examples of necessary truths include truths of mathematics ('2 + 3 = 5') and sentences stating the definition of some expression ('a bachelor is an unmarried man'). Intuitively, these truths are necessary because there is no possible way they could have been false (it is not as though 2 and 3 could have summed to 7, or that some married men could have been bachelors). Relatively uncontroversial examples of necessary falsehoods involve the denials of such claims ('2 + 2 = 11'; 'some bachelors are married men', etc.).

5.1.2 Contingency

Some sentences are neither necessarily true, nor necessarily false. We call sentences like these *contingent sentences*.

> **Contingency**. Sentence S is contingent if and only if S is neither a necessary truth nor a necessary falsehood.

A sentence is contingent when its truth-value could have been otherwise than it actually is. Trump lost the election in 2020, but he could have won; thus, the sentence 'Trump won in 2020' is a contingent falsehood. Biden won in 2020, but he could have lost; thus, the sentence 'Biden won in 2020' is also contingent (it is a contingent truth).

5.2 Properties of sets of sentences

5.2.1 Necessary equivalence

Sometimes the truth-value of one sentence makes a 'demand' on the truth-value of some other sentence. Suppose that Ange is standing to the left of Britt. Then it has to be the case that Britt is standing to the right of Ange. If the first sentence is true, the second sentence has to be true; similarly, if the first sentence is false, the second sentence also has to be false. Sentences like these are called *necessarily equivalent* sentences.

> **Necessary Equivalence**. Sentences S and S' are necesarily equivalent if and only if it is impossible for S and S' to have different truth-values.

Necessarily equivalent sentences could never 'disagree' on their truth-values. Notice that any pair of necessary truths will automatically be necessarily equivalent in this sense. The same goes for any pair of necessary falsehoods.

5.2.2 Joint possibility and impossibility

Two more properties of sets of sentences are those of *joint possibility* and *joint impossibility*.

> **Joint possibility**. A set of sentences $B_1, B_2, \ldots, B_n$ is jointly possible if and only if it is possible for each $B_1, B_2, \ldots, B_n$ to be true together.
>
> **Joint impossibility**. A set of sentences $B_1, B_2, \ldots, B_n$ is jointly impossible if and only if $B_1, B_2, \ldots, B_n$ are not jointly possible.

Joint possibility and impossibility are properties of *sets* of sentences. We use the concept of joint possibility to characterize collections of sentences that could all be true, *together*. Similarly, we use the concept of joint *im*possibility to characterize collections of sentences that could not all be true together. Notice that whenever a set of sentences is jointly impossible, any argument with those sentences as premises is automatically a valid argument (see §2).

5.3 Necessary and sufficient conditions

A final pair of concepts. In stating all of the definitions you have learned so far in the course, we have tacitly made use of the concepts of *necessary* and *sufficient conditions*. Here are the definitions:

> **Necessary Conditions**. B is a *necessary condition* for A whenever it is impossible that A obtains and B does not.
>
> **Sufficient Conditions**. A is a *sufficient condition* for B if A's obtaining is enough to bring it about that B obtains.

These concepts are a bit tricky. Here are some examples to help illustrate the central ideas:

- Necessary conditions:
  - There being oxygen in the room is a necessary condition for there to be a fire in the room.
  - The presence of water is a necessary condition for animal life.
  - A source of electricity is a necessary condition on your being able to charge your smartphone.
  - Your receiving a grade of at least 50% is a necessary condition for you to pass this course.
- Sufficient conditions:
  - Being from Winnipeg is a sufficient condition for being Manitoban.
  - Being born in Winnipeg is a sufficient condition for being a Canadian citizen.
  - A grade of 63% is a sufficient condition for you to pass the course.

Think of a necessary condition as something that has to happen, in order for something else to happen. You could not build a fire without oxygen, since fire *requires* oxygen. Thus the presence of oxygen is a necessary condition for fire.

Think of the sufficient conditions of some event as the various things that are enough, on their own, to bring it about that that event occurs. Suppose I'm wondering what grade I need to pass some course I'm taking. Suppose a passing grade is a 50%. Then I could pass by getting a 50, but I could also pass by getting a 60, or a 70, or an 80, or.... In fact, any grade between 50 and 100 will be enough for me to pass: receiving any grade in that range would be sufficient for me to pass the course.

Philosophers and logicians care a lot about necessary and sufficient conditions. In fact, each of the definitions you have learned so far states a condition that is both necessary and sufficient for something to fall under the concept being defined. That is indicated by the presence of the expression 'if and only if' in each of the definitions. We will discuss the concepts of necessary and sufficient conditions in greater depth in the next module of the course.