# PHIL 1300: Introduction to Logic

## Module One

## Contents



## 1 Overview

Welcome to the course!

This is the first part of a series of course notes intended to supplement your weekly reading and assignments. This set of notes covers material in Chapters 1 and 2 of *For all x: An Introduction to Formal Logic*, which is available to you now on our course website. Chapters 1 and 2 in *Forall x* are the assigned reading for the first module of the course. You should complete that reading before turning to these course notes, which serve primarily to supplement and expand upon the material covered in the text.

## 2 What this course is about

This is an introductory course in *formal logic*. Its aim is to get you thinking in a very precise manner about the nature of good reasoning, and about what distinguishes good reasoning from bad.

- Here is an example of what we mean by 'reasoning'. Suppose I'm wondering whether a Republican will win the American Presidential election in November.[1] Your friend might try to convince you that a Republican will win. And in order to do so, your friend might present you with the following *line of argument* in support of their position:

  > Look, we know that either Biden is going to win, or Trump is. However, Biden isn't going to win unless the Democrats can get out the vote, and if recent history is any guide we know that the Democrats won't be able to do that. Since Trump is a Republican, it follows that Trump is going to win.

  Notice that your friend isn't just uttering a collection of unrelated sentences. Instead, some of those sentences are supposed to be taken as *reasons* for thinking that a Republican is going to win. In fact, the line of reasoning your friend is giving you has a relatively complex structure that we could represent to ourselves as follows:

  1. Either Biden will win, or Trump will win.
  2. Biden won't win unless the Democrats get out the vote.
  3. The Democrats won't get out the vote.
  4. Therefore, Trump will win. (1–3)
  5. Since Trump will win, a Republican will win.
  6. Therefore, a Republican will win. (4, 5)

- What we have above is a list of sentences. But it is not *merely* a list of sentences. The difference between the above and, say, a grocery or to-do list is that in the above example the different sentences in the list play different *roles*.

- Specifically, in the above example, some of the sentences—the 4th and 6th—are intended to be *supported*, or *implied*, or *entailed* by other sentences in the list. We've indicated those relations of support in the parentheses on the right (what the material in parentheses does is show that sentence 4 *follows from* sentences 1–3, and that sentence 6, in turn, *follows from* sentence 4 and sentence 5).

- Whenever a set of sentences has that feature—whenever one of the sentences is intended to follow from, or be supported by, each of the others—we have an example of what philosophers and logicians call an **argument**.[2]

- Here is another example that illustrates the basic pattern:

  1. Trump will win only if substantial progress is made in the fall of 2020 on an effective COVID-19 vaccine.
  2. There won't be substantial progress made during the fall of 2020 on an effective COVID-19 vaccine.
  3. ∴ Trump won't win. (1, 2)[3]

- And here is a third:

[1] These course notes were written in the run-up to the 2020 American Presidential Election.

[2] Notice the difference between that technical sense of 'argument' and the sort of everyday verbal tussles you might engage in with a sibling or parent or friend. Our focus is on the sense of 'argument' that applies to a piece of reasoning intended to support a particular view or opinion (the *conclusion* of the argument; see below).

[3] Following the convention adopted in our textbook, we will use the '∴' notation to abbreviate 'therefore', and cognate expressions like 'it follows that', 'thus', and 'hence'.

1. Trump will win if the majority of Republican voters are willing to overlook his missteps in managing the national response to the Covid-19 pandemic.
2. The majority of Republican voters are are willing to overlook Trump's missteps in managing the national response to the Covid-19 pandemic.
3. ∴ Trump will win. (1, 2)[4]

- Each of the foregoing is an example of what philosophers and logicians mean when they talk about arguments. The following definition isolates what is common to each of the foregoing examples:[5]

> An **argument** is a collection of sentences, one of which is intended to be supported by each of the others.

We're going to spend roughly the first 2/3 of this course thinking very carefully about the nature of arguments. Specifically, we're going to be investigating, in a mathematically precise way, the nature of the *relation* that holds between the parts of an argument whenever that argument counts as a good (or *compelling*, or *rationally persuasive*) argument.

## 3 Truth and falsity

We've been talking a lot so far about *sentences*. An argument is a *collection* of sentences. In an argument, one of those sentences is *supported by* each of the others. Logicians give special names to the sentences that occur in an argument. Here are some more definitions:

> A **conclusion** is any sentence in an argument that is intended to be supported by each of the other sentences in that argument.
>
> A **premise** is any sentence in an argument that is intented to support the argument's conclusion.

Obviously, there is a good question here about just what is meant by 'support'. That's one of the questions we're going to be answering shortly. But for now the important thing is to observe the way the individual sentences in an argument can be *classified* in terms of the above definitions. For example, in the following argument (taken from above), sentences 1 and 2 are functioning as premises, and sentence 3 is functioning as the argument's conclusion:

1. Trump will win only if substantial progress is made in the fall of 2020 on an effective COVID-19 vaccine.
2. There won't be substantial progress made during the fall of 2020 on an effective COVID-19 vaccine.
3. ∴ Trump won't win. (1, 2)

It is important to observe that not just any sentence is capable of serving as a premise or conclusion in an argument. For example, each of the following is a sentence:

- Thank you for dinner.
- Don't mention it.
- Are you sleepy yet?
- Shut the door!!
- If you drink all the beer, buy more.

However, none of the above could serve as a premise or conclusion in any argument. Why not?

- The answer has to do with *truth*. In an argument, the sentences that serve as that argument's premises and conclusion are always capable of being either *true*, or *false*. Another way to put this point is to say that the premises and conclusion of an argument are always *truth-evaluable*. We call sentences that are truth-evaluable **declarative sentences**.
- Lots and lots of sentences are not declarative. For example:
    - Questions ('What time is it?')
    - Imperatives ('Make me a pizza!')
    - Exclamations ('Ouch!', 'Hooray!')
- These sorts of sentences are not capable of serving as premises or conclusions in any argument precisely because they are not capable of being either true or false.

> A **declarative sentence** is a sentence that is capable of being either true or false. Declarative sentences are *truth-evaluable*. Only declarative sentences can serve as the premises or conclusion in an argument.

[4] As it happens, Trump didn't end up winning the 2020 American Presidential Election. But you can ignore that fact for present purposes. What we are focusing on here is the structure of a piece of reasoning, and not every piece of reasoning with the sorts of features we're interested in establishes something true. Sometimes, we consider the reasons for believing something that turns out to be false.

[5] There will be *a lot* of definitions. You should memorize them!

# 4 Consequence and validity

The fact that only declarative sentences are capable as serving as either premises or the conclusion in an argument is central to understanding the notion of **support** alluded to above. This section gets more precise about this central concept of deductive logic.

- Let's begin with the observation that not all arguments are equally strong. Some arguments are such that their premises do a good job of supporting their conclusions, while others lack this feature.
- For example, in the following argument, the premises (if both true) support the conclusion (the conclusion *follows from* the premises, taken together):

  1. If Trump won, then a Republican won.
  2. It isn't the case that a Republican won.
  3. ∴ Trump didn't win. (1, 2)

- Contrast that example with the following:

  1. Any student in *Introductory Logic* that tries their best will do well in the course.
  2. Tom did well in the course.
  3. ∴ Tom tried his best. (1, 2)

- There is an intuitive sense in which the conclusion of this second argument *does not* follow from the two premises taken together. After all, perhaps Tom did well because he had a streak of blind luck on the final, during which he simply guessed at the correct answer to every question.
- We can make this observation more precise in terms of something called *logical consequence*. Here is the definition:

  > Sentence $A$ is a **logical consequence** of a set of sentences $B_1, B_2, \ldots, B_n$ if and only if it is *impossible* that $B_1, B_2, \ldots, B_n$ are true and $A$ is false.

- The basic idea is that a sentence $A$ will be a logical consequence of some sentences $B_1, B_2, \ldots, B_n$ whenever it *could not* be the case that all of the $B$s are true while $A$ is false.
- For a set of premises to support a conclusion is for it to be the case that those premises *entail* that conclusion, which is just for it to be the case that it is impossible for those sentences to be true and the conclusion false.
- Equivalently, for a set of sentences to entail a conclusion is for it to be the case that, necessarily, if the premises are true, the conclusion is also true.

  > A set of sentences $B_1, B_2, \ldots, B_n$ **entail** a conclusion $A$ if and only if $A$ is a logical consequence of $B_1, B_2, \ldots, B_n$

- Here is a bit more notation to help regiment ideas a bit more precisely. Suppose that we write '$B_1, B_2, \ldots, B_n \vdash A$' to represent the fact that sentence $A$ is a logical consequence of sentences $B_1, B_2, \ldots, B_n$. In effect, we are using the '$\vdash$' notation to represent the relation of entailment, or logical consequence.
- We can use the notion of logical consequence to define a further pair of concepts that will be central in the weeks to come. These are the concepts of deductive *validity* and *invalidity*.

  > An argument from a set of premises $B_1, B_2, \ldots, B_n$ to a conclusion $A$ is a **valid argument** if and only if $B_1, B_2, \ldots, B_n \vdash A$.
  >
  > An argument from a set of premises $B_1, B_2, \ldots, B_n$ to a conclusion $A$ is an **invalid argument** if and only if $B_1, B_2, \ldots, B_n \nvdash A$.[a]
  >
  > [a] We are using '$\nvdash$' to represent that fact that $A$ is *not* a logical consequence of $B_1, B_2, \ldots, B_n$.

The basic idea is that for an argument to be valid, its conclusion must be a logical consequence of its premises, which is just to say that it is impossible that the premises be true and that conclusion false. An argument is invalid whenever its conclusion is not a logical consequence of its premises, which is just to say that it is possible for the premises to be true and for the conclusion to be false.

# 5 More on logical consequence

Here is an important clarificatory note on the notion of logical consequence. When we say that a sentence $A$ is a logical consequence of some set of sentences $B_1, B_2, \ldots, B_n$, we are *not* saying the following:

- $B_1 \vdash A$
- $B_2 \vdash A$
- $\vdots$
- $B_n \vdash A$

That is, we are not saying that $A$ is a logical consequence of each of $B_1, \ldots, B_n$ *taken individually*. $B_1, \ldots, B_n$ is a *set* (or collection, or group) of sentences, and what we are saying when we say that $B_1, \ldots, B_n \vdash A$ is that $A$ is a consequence of the entire set (the entire collection of sentences $B_1, \ldots, B_n$, *taken together*). This is an important fact to keep in mind as we talk about validity and some related logical notions in more depth over the course of subsequent modules.

## 6 Practice exercises

The remaining document in this module contains a several practice exercises designed to improve your understanding of the concepts discussed in this handout. You should complete all of those exercises before turning to Module Two.