**Given:**

An integral domain *Z* is a ring for the operations + and \* with three additional properties:

1.  The commutative property of \*: For any elements *x* and *y* in *Z*, *x*\**y*=*y*\**x*.

2.  The unity property: There is an element 1 in *Z* that is the identity for \*, meaning for any *z* in *Z*, *z*\*1=*z*. Also, 1 has to be shown to be different from the identity of +.

3.  The no zero divisors property: For any two elements *a* and *b* in *Z* both different from the identity of +, *a*\**b*≠0.

A field *F* is an integral domain with the additional property that for every element *x* in *F* that is not the identity under +, there is an element *y* in *F* so that *x*\**y*=1 (1 is notation for the unity of an integral domain). The element *y* is called the multiplicative inverse of *x*. Another way to explain this property is that multiplicative inverses exist for every nonzero element.

Modular multiplication, [\*], is defined in terms of integer multiplication by this rule: [*a*]m [\*] [*b*]m = [*a* \* *b*]m

*Note: For ease of writing notation, follow the convention of using just plain \* to represent both [\*] and \*. Be aware that one symbol can be used to represent two different operations (modular multiplication versus integer multiplication).*

A.  Prove that the ring *Z*31 (integers mod 31) is an integral domain by using the definitions given above to prove the following are true:

1.  The commutative property of [\*]

2.  The unity property

3.  The no zero divisors property

B.  Prove that the integral domain *Z*31 (integers mod 31) is a field by using the definition given above to prove the existence of a multiplicative inverse for every nonzero element.

C.  Acknowledge sources, using APA-formatted in-text citations and references, for content that is quoted, paraphrased, or summarized.