1.  The commutative property of [\*]

Let a and b be elements of ring R and and then

(Wilkins 1996)

2.  The unity property

If a and b are both unity, then;

a=a\*b=b

the first equality holding because b is unity and the second equality holding because a is unity. Therefore;

Z\*1=Z

3.  The no zero divisors property

 Let a be a unit in Z with multiplicative inverse a-1

Suppose a\*b=0 for some B є Z

a-1 \*(a\*b)= a-1 \*0; because the multiplication is well defined

(a-1 \* a-1 )\*b= a-1\*0

(a-1 \*a) \*b= a-1 \*0 due to associativity property

1\*b= a-1 \*0 due to multiplicative inverse property i.e. a\* a-1 =1

b= a-1 \*0 multiplication by an identity

b=0

B.  Prove that the integral domain Z31 (integers mod 31) is a field by using the definition given above to prove the existence of a multiplicative inverse for every nonzero element.

Let Z have finitely many elements,

Let a є Z and a≠0.

Since |Z| is finite elements then

of Z cannot all be distinct.

So there are natural numbers k,l Є и such that we obtain then;

Where 1 is the identity of Z. since Z has no zero divisors, and a ≠ 0 we conclude that , which yields;

# References

Wilkins, D. R. 1996. "Mathematics Cours e 111: Alge bra I."