REAL ANALYSIS 1
Midterm, Fall 2016

NAME _____ STUDENT NUMBER _____

Write in complete sentences!!! *Explain* what you are doing and convince me that you understand what you are doing and why. Justify all steps by quoting relevant results from the textbook, class notes, or hypotheses. Do not copy the work of others; **Do your own work!!!** Due Friday, October 21 by 1:30.

5. (Take Home) (a) Show that if $E \in \mathcal{M}$ and $E \subset P$, then $m(E) = 0$. HINT: Let $E_i = E \mathring{+} r_i$, where $\mathbb{Q} = \{r_i\}_{i=1}^{\infty}$. Then $\{E_i\}_{i=1}^{\infty}$ is a disjoint sequence of measurable sets and $m(E_i) = m(E)$. Therefore $\sum m(E_i) = m(\cup E_i) \leq m([0,1))$.

(b) Show that if A is any set with $m^*(A) > 0$, then there is a nonmeasurable set $E \subset A$. HINT: If $A \subset (0,1)$, let $E_i = A \cap P_i$. The measurability of E_i implies $m(E_i) = 0$, while $\sum m^*(E_i) \geq m^*(A) > 0$.