1. Prove that the set *Z31*(integers mod 31) under the operations [+] and [\*] is a ring by using the definitions given above to prove the following are true:

**Closure Property of addition:**

 a,b$\in Z\_{31}$, Using the property of addition we say that ($[a]\_{31}+[b]\_{31}$) =$[a+b]\_{31}$. Then there will be a remainder when $[a+b]\_{31}$ is divided by 31. The remainder will lie within the interval [$z\_{0}, z\_{1}, z\_{2}, z\_{3},…..z\_{30}$]. Hence, [a+b]$ \in Z\_{31}∀ a,b\in Z\_{31}.$

**Closure property of multiplication:**

If a,b$\in Z\_{31}$ then, $([a]\_{31}\*[b]\_{31})$ = $[a\*b]\_{31}$. Then we will get a remainder when $[a\*b]\_{31}$ is divided by 31. The remainder will always lie within the interval [$z\_{0}, z\_{1}, z\_{2}, z\_{3},…..z\_{30}$]. Hence, [a\*b$]\in Z\_{31} ∀$ a,b$\in Z\_{31}$.

**Additive identity property:**

For all of a is an element of $Z\_{31}$ and 0 is an element of $Z\_{31}$ ($a\in Z\_{31}and 0\in Z\_{31})$. Such that:

[a]+0=[a]+0

=[a]+0 by definition of addition

=[a] since 0 is an additive identity of integers

=0+[a]

Which proves that 0 is an identity element for $Z\_{31}$

**Additive Inverse property:**

Since $Z\_{31}$ is a group under addition then all of a is an element of $Z\_{31}$ and -a is an element of $Z\_{31}$ $(a\in Z\_{31} and-a\in Z\_{31})$. Such that:

$$[a]\_{31}+[-a]\_{31}$$

=$[a-a]\_{31}$

=$[0]\_{31}$

=0

=$[-a]\_{31}+[a]\_{31}$

B= 31-a

Thus $[-a]\_{31}$ is the additive inverse of a in $Z\_{31}$

**Associative property of addition:**

If a,b, and c are elements of $Z\_{31}$, $(a,b,c\in Z\_{31})$ then:

$\left(\left[a\right]\_{31}+\left[b\right]\_{31}\right)+[c]\_{31}=\left(\left[a+b\right]\_{31}\right)+[c]\_{31}$ by definition of addition

$=[\left(a+b\right)+c]\_{31}$ by definition of addition

=$[a+\left(b+c\right)]\_{31}$ by associative property of addition

$[a]\_{31}+(\left[b\right]\_{31}+\left[c\right]\_{31})$ by definition of addition

Therefore,$ \left(\left[a\right]\_{31}+\left[b\right]\_{31}\right)+[c]\_{31}=$ $[a]\_{31}+(\left[b\right]\_{31}+\left[c\right]\_{31})$ and a, b, c are elements of $Z\_{31}$.

**Associative property of multiplication:**

If a, b, and c are elements of $Z\_{31}$, $\left(a,b,c\in Z\_{31}\right) $then:

$[a]\_{31}\*\left(\left[b\right]\_{31}\*\left[c\right]\_{31}\right)=[a]\_{31}\*(\left[b\*c\right]\_{31})$ by definition of multiplication

$=[a\*\left(b\*c\right)]\_{31}$ by definition of multiplication

$=[\left(a\*b\right)\*c]\_{31}$ by associative property of multiplication

$=[a\*b]\_{31}\*[c]\_{31}$ by definition of multiplication

$=([a]\_{31}\*[b]\_{31})\*[c]\_{31}$ by definition of multiplication

Therefore, $[a]\_{31}\*\left(\left[b\right]\_{31}\*\left[c\right]\_{31}\right)=([a]\_{31}\*[b]\_{31})\*[c]\_{31}$ and a,b, and c are elements of $Z\_{31}$.

**Commutative Property of addition:**

If a and b are elements of $Z\_{31}$, $\left(a,b\in Z\_{31}\right) $then

$[a]\_{31}+[b]\_{31}=[a+b]\_{31}$ by definition of addition

$=[b+a]\_{31}$ by commutative property of addition

$[b]\_{31}+[a]\_{31}$ by definition of addition

Therefore, $[a]\_{31}+[b]\_{31}=[b]\_{31}+[a]\_{31}$ and all of a and b are elements of $Z\_{31}$.

**Left and right distributive property of addition and multiplication:**

If a, b, and c are elements of $Z\_{31}$, $\left(a,b,c\in Z\_{31}\right) $then:

$[a]\_{31}\*\left(\left[b\right]\_{31}+\left[c\right]\_{31}\right)=[a]\_{31}\*(\left[b+c\right]\_{31})$ by definition of addition

$=[a\*\left(b+c\right)]\_{31}$ by definition of multiplication

$[ab+ac]\_{31}$ by distributive property of integers

$=[ab]\_{31}+[ac]\_{31}$ by definition of addition

$=\left(\left[a\right]\_{31}\*\left[b\right]\_{31}\right)+(\left[a\right]\_{31}\*\left[c\right]\_{31})$ by definition of multiplication

Therefore, $[a]\_{31}\*\left(\left[b\right]\_{31}+\left[c\right]\_{31}\right)=\left(\left[a\right]\_{31}\*\left[b\right]\_{31}\right)+(\left[a\right]\_{31}\*\left[c\right]\_{31})$ the left distributive property holds when a, b, and c are elements of $Z\_{31}$

Also,

$\left(\left[b\right]\_{31}+\left[c\right]\_{31}\right)\*[a]\_{31}=(\left[b+c\right]\_{31}\*[a]\_{31}$ by definition of addition

$=[\left(b+c\right)\*a]\_{31}$ by definition of multiplication

$=[ba+ca]\_{31}$ by distributive property of integers

$=[ba]\_{31}+[ca]\_{31}$ by definition of addition

$=\left(\left[b\right]\_{31}\*\left[a\right]\_{31}\right)+(\left[c\right]\_{31}\*\left[a\right]\_{31})$ by definition of multiplication

Therefore, $\left(\left[b\right]\_{31}+\left[c\right]\_{31}\right)\*[a]\_{31}=\left(\left[b\right]\_{31}\*\left[a\right]\_{31}\right)+(\left[c\right]\_{31}\*\left[a\right]\_{31})$ when a, b, and c are all elements of $Z\_{31}$ and the right distributive property holds.