1. Prove that the set *Z31*(integers mod 31) under the operations [+] and [\*] is a ring by using the definitions given above to prove the following are true:

**Closure Property of addition:**

a,b, Using the property of addition we say that () =. Then there will be a remainder when is divided by 31. The remainder will lie within the interval []. Hence, [a+b]

**Closure property of multiplication:**

If a,b then, = . Then we will get a remainder when is divided by 31. The remainder will always lie within the interval []. Hence, [a\*b a,b.

**Additive identity property:**

For all of a is an element of and 0 is an element of (. Such that:

[a]+0=[a]+0

=[a]+0 by definition of addition

=[a] since 0 is an additive identity of integers

=0+[a]

Which proves that 0 is an identity element for

**Additive Inverse property:**

Since is a group under addition then all of a is an element of and -a is an element of . Such that:

=

=

=0

=

B= 31-a

Thus is the additive inverse of a in

**Associative property of addition:**

If a,b, and c are elements of , then:

by definition of addition

by definition of addition

= by associative property of addition

by definition of addition

Therefore, and a, b, c are elements of .

**Associative property of multiplication:**

If a, b, and c are elements of , then:

by definition of multiplication

by definition of multiplication

by associative property of multiplication

by definition of multiplication

by definition of multiplication

Therefore, and a,b, and c are elements of .

**Commutative Property of addition:**

If a and b are elements of , then

by definition of addition

by commutative property of addition

by definition of addition

Therefore, and all of a and b are elements of .

**Left and right distributive property of addition and multiplication:**

If a, b, and c are elements of , then:

by definition of addition

by definition of multiplication

by distributive property of integers

by definition of addition

by definition of multiplication

Therefore, the left distributive property holds when a, b, and c are elements of

Also,

by definition of addition

by definition of multiplication

by distributive property of integers

by definition of addition

by definition of multiplication

Therefore, when a, b, and c are all elements of and the right distributive property holds.