The Mesopotamian culture is often called Babylonian, after the large metropolis of that name. We could “babble on”[**1**](https://jigsaw.vitalsource.com/books/9781323104927/content/id/ch02fn1) and on about their many fine achievements in architecture, irrigation, and commerce, but it is their mathematics that is truly remarkable, dwarfing that of other contemporary civilizations. One might not be impressed by their use of a vertical mark for “one” and a horizontal mark for “ten” – ten being a common unit in the mathematics of many societies, including Egypt, China, Rome, and our own society today. On the other hand, they were the first to employ a “positional” system which, except for minor changes, survives to this day!

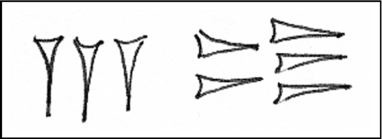
1The authors would like to apologize for the easy pun, but we couldn’t resist.

Let’s remind ourselves how our current number system works. It does not suffice to say that it is based on grouping by tens. The Egyptians did this – yet we have left them in the dust by taking a giant step forward to the “position system.” We require only ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Nevertheless, we can handle numbers of any size without the need to define a new symbol. This is because the value of a number is determined not just by the symbol. We must note the *position*of the symbol as well. The two 3’s in the number 373 represent different quantities. You would rather have three hundred dollars than three dollars, right? To summarize, our number system employs a mere ten symbols, whose values depend on their position in the number. Moving one digit to the left multiplies its place value by ten, while moving to the right (not surprisingly) divides its place value by ten.

Observe, by the way, that this is true on both sides of the decimal point! In the number 3.1416, the 1 near the 6 is worth only one hundredth of the 1 near the 3. There is no number in the entire universe that is too large or too small for our clever (ten-digit!) number system (of Hindu-Arabic origin, by the way). We call our system the *decimal* system, because ten is the base.

The Babylonians used instead the *sexagesimal* system because they chose 60 as their base. While we are not sure why, we are fairly certain they did not have 60 fingers. One theory (which is very popular) is that 60 has a multitude of factors, that is, many numbers go into 60. Put another way, $60 can be divided without coin among 2, 3, 4, 5, 6, 10, 12, 15, 20, or 30 people. We shall follow the common practice of using commas to separate groups. Thus (3, 50)60 shall mean 3 sixties and 50 ones for a total of 230. What does (2, 3, 50)60 mean? Well in our position system, 357 means 3 hundreds, 5 tens, and 7 ones, right? Each column is ten times more valuable than its neighbor. In the same way, each column to the left in the Babylonian system is sixty times bigger! In the number (2, 3, 50)60, the 2 represents 2 3600’s – because 60 × 60 = 3600. The next column to the left would represent 60 × 3600 or 216000.

The Babylonians only used two symbols: a vertical mark for 1 and a horizontal mark for 10. Thus, the number 230, which we denoted by (3, 50)60 would look like this:



Addition and subtraction were done by working in columns and often required “borrowing” (just as the Egyptians required). Once they figured out their position system, they took another giant step. They treated fractions as a continuation of the base-60 system! A 1 in the column to the right of the units column has the value 1/60, while a 1 in the next column has the value 1/3600, and so on. Babylonian fractions were so accurate and easy to use that the Greeks, whose number system for whole numbers was markedly different, employed them a thousand years later.

Unfortunately, the Babylonians neglected to do two things:

* 1. They had no symbol for zero, no placeholder to indicate that a column had no entry. This is similar to my writing 301 as 31. After all since there are no “tens” in 301, why put the zero there? The obvious answer is that without the zero, we will conclude that the “3” represents tens. The Babylonians had to use context and guesswork to read their numbers correctly!
* 2. Another “small” oversight: They had no decimal point (or sexagesimal point?). Imagine leaving out the decimal point in the price of a $17.95 steak and scaring away all your customers. The decimal point lets us know where the fractions begin! This problem, too, required caution in the way the Babylonians read their numbers.

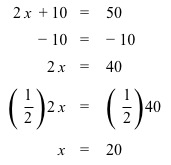
We shall use a semicolon to separate the whole number columns from the fractional ones (like the decimal point of today’s base-ten arithmetic).

The Babylonians multiplied much the way we do, including the use of “carrying,” but their division was noteworthy. They used a table of reciprocals. (See [**Table I**](https://jigsaw.vitalsource.com/books/9781323104927/content/id/ch02tab1).) Recall that the reciprocal of *n* is 1/n, which has the effect of turning it upside-down. The reciprocal of 2/5 is 5/2, for example. Now, since 2 can be written 2/1, its reciprocal is 1/2. The reciprocal of 4 is 1/4. Why the interest in reciprocals? Because division by a number *n* is the same as multiplication by its reciprocal 1/n. To divide by four, one can multiply by one-fourth! Herein lies one of the strengths of the base-sixty system. Many fractional parts of 60 are whole numbers. One half of 60 is 30. So ½=30/60 or (0; 30)60 in the way we have been writing Babylonian numbers. One-third of 60 is 20, so 1/3=20/60 or (0; 20)60.

The Babylonians divided their daylight into twelve hours as the Egyptians did. Their fixation on sixty manifested itself in their dividing the hour into 60 minutes (“minute” for “small”) and the minute into 60 seconds (the “second” division of the hour). So our clocks are Babylonian and Egyptian. It is strange that this practice persists in the decimal age! (Perhaps it isn’t so strange – Americans still resist the “metric” system, preferring 12 inches in a foot, 3 feet in a yard, etc.)

The Babylonians developed their commercial mathematics and computed interest tables for financing loans! They extracted square and cube roots and tackled simple “word problems” requiring the solution of quadratic equations. While their geometry sufficed, they were particularly skilled in algebra and computing with fractions. Their algebra, however, lacked the notation that facilitates ours, that is, “let *x* be Johnny’s age, and *x* + 4 be Sally’s age. Then in two years, the sum of their ages will be (*x* + 2) + (*x* + 6) = 2*x* + 8.” Rather, theirs was a procedural algebra. Thus, if twice a quantity of apples augmented by ten apples yields fifty, then a Babylonian might tell you to “taketh away ten apples from the fifty and then ye must take half the remaining quantity. Then by the gods, thou shalt be left with the original amount.”

Of course this is exactly what we do today:



Babylonian cosmology was a mixture of science and religion. It was believed that the gods resided on the planets, from which it followed that their location would influence the actions of the gods in human affairs – hence, the birth of astrology. The Greek and Roman beliefs in this regard are Babylonian in origin. The Babylonians assumed that Earth was the center of the universe and everything revolved around it. This cosmology was later adopted by the early Christian thinkers and perpetuated for a thousand years after the fall of Rome. The “geocentric theory” was consistent with the Christian belief that the Son of God was born at the center of the universe.

Among the many tablets dug up in the Fertile Crescent, one of them, called *Plimpton 322*, contained many pairs of numbers with a curious property. The square root of the difference of their squares is a whole number. You might recall the famous 3, 4, 5 triple that has this property, that is, 32 + 42 = 52. We will meet such triples when we study the Greek mathematician, philosopher, and cult figure Pythagoras and his famous theorem which says that the sum of the squares of the two shorter sides of a right triangle equals the square of the hypotenuse, or as it is usually presented,

*a*2 + *b*2 = *c*2

where *a* and *b* represent the lengths of the shorter sides (called the *legs*) and *c* is the length of the hypotenuse.

Now things usually get messy when the two shorter sides (or “legs”) are randomly selected. If they are 2 and 5, for example, then the sum of their squares is 29 (i.e., 4 + 25 = 29), implying that the hypotenuse is. Without a calculator, this poses a significant challenge, though it’s easy to say it is between 5 and 6. You might also recall the triple 5, 12, 13 – an amazing triple. While these triples might result from a lucky guess or two, the triples on the recovered clay tablet go into the thousands, for example 2291, 2700, and 3541, indicating that the Babylonians had a method probably similar to that of the Greeks.



We close this chapter with an amazing tale of cleverness. The Babylonians managed to compute square roots quite accurately, as is exemplified by their calculating  to seven places of accuracy. We surmise that they used a method of averaging that goes something like this. Suppose we want . Let us begin with a guess of 15. This is obviously too small since 15 × 15 = 225, whereas the square root of a number times itself should yield the original number, in this case 363. Furthermore, if we calculate 363 ÷ 15, we get 24.2 (which is too big to be ). Well here is a brilliant idea. Since 15 is too small and 24.2 is too big, let’s take their average, that is, half of their sum, or (24.2 + 15)/2, which is 19.6. This is much closer to the answer than 15 or 24.2. Let’s repeat this procedure one more time. Since 363/19.6 = 18.520408163 … ≅ 18.52, and once again 18.52 is too small and 19.6 is too big, let’s average them: (18.52 + 19.6)/2 = 19.06, which is even closer to the “true” answer of 19.05255888325 . . . . This *algorithm* (procedure) can be repeated as often as one wants, yielding a sequence of increasingly accurate answers.



Another way of calculating square roots is to use the formula



where *N* = *S*2 + *E*, that is, *S* is the largest number whose square is less than *N* and *E* is the difference between this perfect square and *N*. While the Babylonians didn’t have a formula, don’t forget they had a procedural algebra that could describe this formula.

In summary, Babylonian mathematics was extremely sophisticated for its time. Our position system today is virtually identical to theirs, as is our system of time measurement. The next great ancient civilization we study, Classical Greece, inherited Babylonian algebra and Egyptian geometry, no doubt, through the travels of her scholars and merchants. It should be noted that very little is known about the mathematics of ancient China. In 213 B.C., the emperor Shi Huang of the Chin dynasty had all of the manuscripts of the kingdom burned. Fortunately, a copy of a work titled*Arithmetic in Nine Sections* survived. Written before 1000 B.C., it contains mathematics roughly on par with that of Egypt and Babylon.