**Greeks Bearing Gifts**

A great concern among Greek philosophers, most notably Plato and Aristotle, was the meaning of universals, or forms. When a person uses the word dog, to what do they refer? In the famous quote, “Outside of a dog, a book is man’s best friend. Inside of a dog, it’s too dark to read…” the form or universal dog is being discussed. Some dogs are tall, others short. They occur in a variety of colors. Some are friendlier than others. Yet they all share something or they wouldn’t be dogs. The Greek philosophers were interested in the abiding essence of dog.[**1**](https://jigsaw.vitalsource.com/books/9781323104927/content/id/ch04fn1) After all, every dog on this planet will eventually die – but “dogness” will go on forever, we hope. This is called the problem of universals in philosophy and it has many different answers.

The famous fourth-century (B.C.) philosopher Plato believed that there is a perfect dog which resides in the world of universals – a mystical place where our immortal souls come from prior to our birth. This universal dog is stripped of all of its nondog-specific characteristics. It has no specific color or height or weight. It wouldn’t be your pet, Fango, because he is a specific dog, while the universal dog has only general characteristics essential to every dog. Plato’s perfect puppy will never die – and it doesn’t have to be walked. It is truly the ideal dog. In fact, the term “ideal” comes from Plato’s world. The forms there give us the “idea” of dog, cat, man, or whatever class of things we have the idea of. The forms are eternal and worthy of our study, whereas earthly objects are imperfect shadows of the forms. You may have read Plato’s cave analogy in his book *The Republic*, in which he expounds on this theme. It should come as no surprise that Plato loved geometry. Geometry establishes necessary connections between the forms of the polygons we see around us. We don’t speak of *this* rectangular table or *that* circular clock on the wall in geometry. We discover truths, rather, of the perfect circle or the perfect rectangle. These are eternal truths.

Plato founded a school of philosophy which he named the Academy, and legend has it that he wrote over its portals “Let none enter here who have not studied geometry.” Geometric truths, thought Plato, are apprehended through pure reason, unsullied by perception. The diagrams simply allow us to recollect things our souls knew in that other world (of universals). A legacy of his is “Platonic Love” – pure and chaste – a love of the eternal soul of the other person unblemished by carnal (and temporary) lust for the flesh. You can imagine that Plato wasn’t the life of the party.

Aristotle, a student of Plato’s, departed from all of this. He rejected the world of universals, maintaining that the essence of dog resides in each dog. Perception, without reason, however, will not reveal the essence. Rather, we form the idea of dog by inducing it from observation of many particular dogs.

Their differences aside, these great philosophers, including Plato’s teacher Socrates, secured the firm belief in logical structure employing classes with certain properties. Once we establish that something is a member of a class, we may assume that it has all of the properties of that class. So if Fango is a dog, then he barks, wags his tail, and chases chariots.

Aristotle posited that knowledge begins with certain basic irrefutable assumptions or axioms, without which nothing can be said. If one were to say to him, “You must prove everything – even your most basic assumptions,” he would have disagreed by showing you an absurd “*infinite regress*” that would result. Statement *A* would depend on *B*, which would depend on *C*, and so on, to infinity, and nothing would be proven!

Aristotle, by the way, was fascinated by infinity and wrote that a line segment is infinitely divisible, disagreeing with the views of pre-Socratics like Zeno and Democritus who believed that everything has an ultimate building block or atom which is indivisible. Zeno stated several paradoxes of motion resulting from his belief that a moving object travels from one point of its trajectory to the next. We know today that between any two points on a line, there are infinitely many others.

The logical fondations of mathematics would have to wait for the great Euclid of Alexandria. Why Alexandria if he was Greek? Alexander the Great (the great what?) conquered an enormous territory including Greece, the Balkans, Turkey, Egypt, the Middle East, Persia, and so forth, extending all the way to northern India. He founded a city named Alexandria where a great Museum flourished. It contained a library of over 100,000 Greek manuscripts. Upon Alexander’s death in 323 B.C., his vast kingdom was split into three empires, one being Egypt. Though the Romans conquered Egypt in the first century B.C., they permitted Greek culture, including the Museum, to flourish. The library was finally burned to the ground in 641 A.D. by Islamic invaders. The word “museum” derives from the Muses, the nine sisters of the arts, from which “music” is also obtained.

Euclid[**2**](https://jigsaw.vitalsource.com/books/9781323104927/content/id/ch04fn2) flourished around 300 B.C. and lectured at the Museum. He wrote a thirteen-volume work called *The Elements*, which has appeared in more editions than any other book except for the Bible.

2When Euclid was asked by a student, “What shall I gain by learning these thing?” he replied, “Someone give him a penny, since he must make a profit from the things he learns.”

*The Elements* summarizes 300 years of Greek geometry and number theory – but it does much more than that! Euclid establishes definitions. A subject must start by defining the things it studies. A point has location without extension. A line is the shortest distance between two points. Parallel lines don’t meet no matter how far they are extended. A triangle is a three-sided figure, and so forth and so forth.

He then presents common notions and axioms, such as “equals added to equals yield equals” and “the whole is greater than its parts.” He assumes, for example, that through a point not on a given line, there exists a unique (only one) line parallel to the given line, though he states this in a very roundabout way. This “parallel postulate” has been doted on for centuries until, in the nineteenth century, new geometries were founded without this postulate. They are called *non-Euclidean* and play an important role in the theory of relativity.

Euclid’s volumes were so well done and summarized earlier works so thoroughly that it made older works obsolete. Later-day Islamic translators felt it unnecessary to translate these works, resulting in the tragic loss of many earlier manuscripts.

Euclid was interested in prime numbers and proved that there are infinitely many of them. In case you think this is obvious, consider this. After the prime number 2, no even number is prime. This eliminates half the numbers. After 3, every third number is a multiple of three and, therefore, not a prime. After 5, every fifth number fails to be prime, and one sees that it seems quite conceivable that at some point there are no more primes. Before we look at Euclid’s proof of the infinitude of primes, we need some preliminary observations:

1. Every integer can be factored down to prime factors. Let’s look at an example of this fact.

2. A number is divisible by another number if and only if the second number is a factor of the first. Thus, 36 is divisible by 12 because 36 = 3 × 12.

3. If a number, say *n*, is divisible by another, say *m (m* > 1), then if we add one to the first number, it is no longer divisible by the second, that is, *n* + 1 is **not** divisible by *m*.

4. A (rather lengthy) product of many integers is divisible by each of them. For example, 2 × 3 × 5 × 7 = 210 which is divisible by each of 2, 3, 5, and 7. (It is divisible by more numbers such as 6 and 10.)

5. Finally, if a number > 1 is composite (not prime), it must have a prime factor.

Euclid began his proof by assuming that there are only finitely many primes and hoped to obtain a contradiction. If there are finitely many primes, let us call them *a, b, c*, and so on, up to the last (or greatest) prime, say, *p*. Then any number larger than *p* is, by our assumption, a composite number – because we are assuming that *p* is the biggest prime. Now, said Euclid, consider the number *a* × *b*× *c* × … × *p* called *N*, and finally consider the even larger number *N* + 1. Since this number is obviously bigger than *p*, it is a composite number. Then it must have a prime factor. This prime is clearly a factor of *N* and therefore goes into *N*. It follows, then, that this prime factor does not go into *N* + 1. This is an enormous contradiction. *N* + 1 is a composite number without a prime factor! If you don’t get it, read the proof again – it’s worth it. This is one of the greatest proofs of antiquity.[**3**](https://jigsaw.vitalsource.com/books/9781323104927/content/id/ch04fn3)It is even more amazing to us that Greek scholars asked the question, “are there infinitely many primes?” – this question has very little bearing on the price of tea in Babylon. Once again, we see that the Greeks loved knowledge for its own sake and not for its technological benefits.

Euclid developed an algorithm (procedure) for determining the *greatest common divisor*, henceforth abbreviated GCD, of two integers. Reminder: The GCD of 15 and 20 is 5 because 5 is the greatest number which goes into both 15 and 20. It is easy to figure out the GCD of small numbers by guesswork, but what do we do if the numbers are large? Euclid began with the following assumption. If a number *n* goes into *x* and into *y*, then it also goes into *x − y*. He then reasoned that we could subtract the smaller number from the larger as many times as possible and then compute the GCD of the remainder and the smaller number. This yields a much easier problem, after which the procedure may be repeated.