# **MATERIAL FOR FINAL EXAM**

This list of the material for the Final Exam is in no particular order. It is the material that occurred to me as I was making out the Final Exam. Some of the topics can actually be listed in more than one area. I tried to place each topic where it was most natural.

Remember, no one single version of the Final Exam can have all of this material. It is simply a matter of "the luck of the draw" as to which version of the Final Exam you get. There are several versions of the Final Exam being given.

For the date and time of the Final Exam see the separate document titled Information For Final Exam.

Here is the list:

### • Basic Concepts

- Opposites of numbers
- Reciprocals of numbers
- Absolute values of numbers
- Order of operations
- Radicals
- Combining similar terms
- o Sets
  - Rule Method •
  - Roster Method

# • Classifying Numbers

- $\circ$  Natural numbers: 1, 2, 3, 4, 5, ...
- Whole numbers: 0, 1, 2, 3, 4, 5, ...
- Integers: ...-3, -2, -1, 0, 1, 2, 3, ...
- Rational numbers:
  - integers
  - common fractions
  - mixed numbers
  - terminating decimals
  - repeating decimals
- Irrational numbers:
  - most radicals
  - stuff with  $\pi$ •
  - stuff with *e*
  - look, funny-looking decimals that do not repeat and do not terminate
- Real numbers: the rationals and irrationals together.

### Exponents

• Laws of Exponents

• 
$$a^{m} \times a^{n} = a^{m+n}$$
  
•  $\frac{a^{m}}{a^{n}} = a^{m-n}$ 

- $(a^m)^n = a^{m n}$
- Negative exponents
- Anything to the zero power, except for zero, is 1
- Scientific notation

## • Factoring

- Common expressions
- o Grouping
  - Group first two and last two.
  - Group first three and last one. Always leads to a difference of squares.
- o Difference of squares
- Sums and differences of cubes
- Factoring Trinomials
  - Splitting the middle term

## • Equations

- Linear equations
  - 0 = 0 means solution is all real numbers
  - 0 = something other than 0 means no solution
- Absolute value equations
  - | X | = p, where p is positive is equivalent to solving X = -p or X = p. You get two different answers.
  - |X| = n, where n is negative has no solution.
- Quadratic equations
  - Factoring
  - Square root principle
  - Quadratic formula
- Rational equations
  - Remember to check for extraneous solutions. A solution is extraneous if it makes one or more denominator equal to zero in the original equation
- o Radical equations
  - Remember to check for extraneous solutions. A solution is extraneous if changing one or more signs in the original equation would make it work.

# • Inequalities

- Linear inequalities
  - Reverse the direction of the inequality if you multiply or divide by a negative number.
- Absolute value less than
  - |X| < p where p is positive is equivalent to solving -p < X < p
  - | X | < n where n is negative has no solution
- Absolute value greater than
  - | X | > p where p is positive is equivalent to solving X < -p or X > p. You will get two separate intervals
  - | X | > n where n is negative has solution all real numbers

- o "and" and "or" inequalities
- Three-sided inequalities
  - The idea is to isolate the variable in the middle
- Interval notation
  - Always put the smaller number on the left.
  - For the following, assume that *a* < *b*:
    - a < x < b is (a, b)
    - $a \le x \le b$  is [a, b]
    - $a < x \le b$  is (a, b]
    - $a \le x < b$  is [a, b]
    - x > a is  $(a, \infty)$
    - $x \ge a$  is  $[a, \infty)$
    - x < b is  $(-\infty, b)$
    - $x \le b$  is  $(-\infty, b]$
    - All real numbers is  $(-\infty, \infty)$

# • Word Problems

- Number problems
- Consecutive numbers
  - Look like *x*, *x* + 1, *x* + 2, *x* + 3, ...
- o Consecutive even/odd numbers
  - Look like *x*, *x* + 2, *x* + 4, *x* + 6, ...
- Rope problem
- Interest problems
- o Rectangle and area or perimeter
- Max/min problems
  - You are looking for the vertex of a parabola
  - 1<sup>st</sup> coordinate of the vertex is where or when or what causes the max/min value
  - 2<sup>nd</sup> coordinate of the vertex is what the max/min value is
- Rocket scientist
- o Triangle
  - The sum of the three angles is 180°.
- Pythagorean Theorem problems
- Flower bed, swimming pool, picture frame
- o Motion problem
  - Use d = rt (distance = rate × time. Rate is speed)
- o Work
  - Mowing a lawn
  - Painting a house
  - Filling a tank or reservoir or sink
- o Stay within budget
- o Ski trip problem
- o Linear function word problems
- Mixture problems

#### • Lines

- Slope:  $m = \frac{y_2 y_1}{x_2 x_1}$ 
  - Slope of a vertical line is undefined

• Two-point form: 
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

- Point-slope form:  $y y_1 = m(x x_1)$
- Slope-intercept form: y = mx + b
- General form: Ax + By = C where A, B, and C are integers with not both A and B zero at the same time and  $A \ge 0$ .
- o Parallel lines
  - Same slopes but different *y*-intercepts
- Perpendicular lines
  - Slopes are negative reciprocals of each other
- Vertical and horizontal lines
  - Simplest form of a vertical line is *x* = a number
  - Simplest form of a horizontal line is *y* = a number
- o Graphs
- o Intercepts

### • Functions and Relations

- o Domain
- o Range
- Evaluating functions
- Difference quotient

$$\frac{f(x+h) - f(x)}{h}$$

$$\frac{f(x) - f(a)}{x - a}$$

- Graphs
- Piecewise functions
- o Intercepts
- Vertical line test
- o Zeros

# • Quadratic Functions and Equations

- Quadratic formula
- General form:  $f(x) = ax^2 + bx + c$
- Standard form:  $f(x) = a(x-h)^2 + k$

• 
$$h = \frac{-b}{2a}$$

- k = f(h)
- Vertex is at V(h, k)
- Axis of symmetry is x = h
- Minimum value of k if a > 0
- Maximum value of k if a < 0

- *h* is where or when the max/min value occurs
- *k* is what the max/min value is
- Nature of solutions and discriminant
  - Discriminant =  $b^2 4ac$
  - If discriminant is positive, then two distinct real solutions
  - If discriminant is negative, then zero real solutions
  - If discriminant is zero, then one real solution.
  - *k* problems
- o Zeros

### • Rational Expressions (commonly called fractions)

- Adding and subtracting
- Multiplying and dividing
- Complex fractions

## • Circles

- Equation:  $(x h)^2 + (y k)^2 = r^2$
- Center is C(h, k)
- Radius is r
- Diameter is twice the radius

## • Symmetry

- Symmetry with respect to the *x*-axis
  - Replace y with -y. If equivalent to the original equation, then the graph is symmetric with respect to the x-axis, otherwise it is not.
  - If folding the graph along the *x*-axis has pieces matching up, then the graph is symmetric with respect to the *x*-axis, otherwise it is not.
- Symmetry with respect to the y-axis
  - Replace x with -x. If equivalent to the original equation, then the graph is symmetric with respect to the y-axis, otherwise it is not.
  - If folding the graph along the *y*-axis has pieces matching up, then the graph is symmetric with respect to the *y*-axis, otherwise it is not.
- Symmetry with respect to the origin
  - Replace x with -x and y with -y. If equivalent to the original equation, then the graph is symmetric with respect to the origin, otherwise it is not.
  - If rotating the graph through 180° is the same graph, then the graph is symmetric with respect to the origin, otherwise it is not.
- Even and Odd Functions
  - Even functions are symmetric with respect to the *y*-axis.
  - Odd functions are symmetric with respect to the origin.
  - A function that is not symmetric with respect to neither the *y*-axis nor origin is neither even nor odd.
  - If f(-x) = f(x), then the function is even.
  - If f(-x) = -f(x), then the function is odd.

- If f (-x) does not equal f (x) or -f (x), then the function is neither even nor odd.
- Translations and Transformations
  - $\circ$  Translations
    - Vertical translations
      - f(x) + k is a vertical translation by k-units. If k > 0, go up. If k
        < 0, go down,</li>
    - Horizontal translations
      - f (x + h) is a horizontal translation by h-units. If h > 0, go left. If h < 0, go right.</li>
    - Combining vertical and horizontal translations
  - Transformations: Stretching, shrinking, and reflecting
    - Vertical transformations
      - *af* (*x*) is a vertical stretch/shrink by a factor of *k*. Multiply all *y*-values by *a*.
    - Horizontal transformations
      - *f* (*bx*) is a horizontal stretch/shrink by a factor of *h*. Divide all *x*-values by *h*.
  - Combining translations and transformations
    - Given af(bx + h) + k start inside the parentheses and work out:
      - Horizontal translation
      - Horizontal stretch/shrink/reflection
      - Vertical stretch/shrink/reflection
      - Vertical translation

### • Distance and Midpoint Formulas

- Distance formula:  $\sqrt{(x_2 x_1)^2 + (y_2 y_1)^2}$
- Midpoint formula:  $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$