HOMEWORK SET NO. 3

Problem 1.

In this problem you will derive an expression similar to the Child-Langmuir Law but modified for the condition where the initial ion velocity entering the sheath $V_{io} = V_i(x=0)$ is not zero. We will still include the presheath potential drop in the overall potential drop between the plasma and the wall (i.e. $\Phi = \phi_o - \phi_w$), but the Bohm velocity is negligible compared to the injection velocity for this problem, i.e. $V_{io} \gg U_B$.

a) Derive the correct form of Poisson's Equation for this situation, i.e. an expression analogous to Eq. (3) in Notes Set #7.

b) Rewrite Poisson's Equation from part a) in non-dimensional form in which the variables are non-dimensionalized as follows:

$$\chi = \frac{\phi}{\Phi}, \quad \xi = \frac{x}{d}, \quad \chi_o = \frac{\phi_o}{\Phi}, \quad \tilde{V}_i = \frac{V_i}{\sqrt{2e\Phi / m_i}}, \quad \tilde{V}_{io} = \frac{V_{io}}{\sqrt{2e\Phi / m_i}}$$

c) Use your equation from part b) to find the nondimensional electric field at the sheath edge, i.e. $\left.\frac{d\chi}{d\xi}\right|_{\xi=0}$ Hint: for the case we are considering with non-zero ion injection velocity, the potential will pass through a maximum inside the sheath corresponding to (from conservation of energy):

$$e\phi_{\max} = e\phi_o + \frac{m_i V_{io}^2}{2} \ .$$

d) Use your result from part c) in your equation from part b) to find a relation between the total potential drop and the ion current density through the sheath. Express your equation back in dimensional form, $J_i = J_i(V, d, V_{io})$. Simplify as much as possible, but it will not be as "simple" an expression as we found for the C-L sheath.

e) Confirm that your answer from part d) reduces to the C-L result when $V_{io} = 0$.

Problem 2.

A 2-mm by 2-mm square probe is immersed in a 3 eV xenon plasma. The ions are at room temperature, 298 K.

a. If the probe collects 1 mA of ion current, what is the plasma density? Hint: The probe collects current on both sides and can be assumed infinitely thin.

b. What is the floating potential?

c. If the probe is biased to the plasma potential, what is the (net) current collected? Note for this situation, the ions (Maxwellian) can be assumed to have their *most probable speed*, $\sqrt{2kT_i / m_i}$, sometimes also referred to as the *thermal speed*.