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APPENDIX A: A Spreadsheet Model for Efficient Diversification

Several software packages can be used to generate the efficient frontier. We will demonstrate the method using Microsoft Excel. Excel is far from the best program for this purpose and is limited in the number of assets it can handle, but working through a simple portfolio optimizer in Excel can illustrate concretely the nature of the calculations used in more sophisticated “black-box” programs. You will find that even in Excel, the computation of the efficient frontier is fairly easy.

We apply the Markowitz portfolio optimization program to a practical problem of international diversification. We take the perspective of a portfolio manager serving U.S. clients, who wishes to construct for the next year an optimal risky portfolio of large stocks in the U.S. and six developed capital markets (Japan, Germany, U.K., France, Canada, and Australia). First we describe the input list: forecasts of risk premiums and the covariance matrix. Next, we describe Excel’s Solver, and finally we show the solution to the manager’s problem.

The Covariance Matrix

To capture recent risk parameters the manager compiles an array of 60 recent monthly (annualized) rates of return, as well as the monthly T-bill rates for the same period.

The standard deviations of excess returns are shown in Spreadsheet 7A.1 (column C). They range from 14.93% (U.K. large stocks) to 22.7% (Germany). For perspective on how these parameters can change over time, standard deviations for the period 1991–2000 are also shown (column B). In addition, we present the correlation coefficient between large stocks in the six foreign markets with U.S. large stocks for the same two periods. Here we see that correlations are higher in the more recent period, consistent with the process of globalization.

The covariance matrix shown in Spreadsheet 7A.2 was estimated from the array of 60 returns of the seven countries using the COVARIANCE function from the dialog box of *Data Analysis* in Excel’s Tools menu. Due to a quirk in the Excel software, the covariance

	A	B	C	D	E	F	G	H
1								
2								
3	7A.1 Country Index Statistics and Forecasts of Excess Returns							
4	Country	Standard Deviation		Correlation with the U.S.		Average Excess Return		Forecast
5		1991-2000	2001-2005	1991-2000	2001-2005	1991-2000	2001-2005	2006
6	US	0.1295	0.1495	1	1	0.1108	-0.0148	0.0600
7	UK	0.1466	0.1493	0.64	0.83	0.0536	0.0094	0.0530
8	France	0.1741	0.2008	0.54	0.83	0.0837	0.0247	0.0700
9	Germany	0.1538	0.2270	0.53	0.85	0.0473	0.0209	0.0800
10	Australia	0.1808	0.1617	0.52	0.81	0.0468	0.1225	0.0580
11	Japan	0.2432	0.1878	0.41	0.43	-0.0177	0.0398	0.0450
12	Canada	0.1687	0.1727	0.72	0.79	0.0727	0.1009	0.0590

	A	B	C	D	E	F	G	H	I
13									
14	7A.2 The Bordered Covariance Matrix								
15	Portfolio								
16	Weights		1.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
17			US	UK	France	Germany	Australia	Japan	Canada
18	1.0000	US	0.0224	0.0184	0.0250	0.0288	0.0195	0.0121	0.0205
19	0.0000	UK	0.0184	0.0223	0.0275	0.0299	0.0204	0.0124	0.0206
20	0.0000	France	0.0250	0.0275	0.0403	0.0438	0.0259	0.0177	0.0273
21	0.0000	Germany	0.0288	0.0299	0.0438	0.0515	0.0301	0.0183	0.0305
22	0.0000	Australia	0.0195	0.0204	0.0259	0.0301	0.0261	0.0147	0.0234
23	0.0000	Japan	0.0121	0.0124	0.0177	0.0183	0.0147	0.0353	0.0158
24	0.0000	Canada	0.0205	0.0206	0.0273	0.0305	0.0234	0.0158	0.0298
25	1.0000		0.0224	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
26	0.0600	Mean							
27	0.1495	SD							
28	0.4013	Slope							
29									
30	Cell A18 - A24	A18 is set arbitrarily to 1 while A19 to A24 are set to 0							
31	Formula in cell C16	=A18 ... Formula in cell I16 =A24							
32	Formula in cell A25	=SUM(A18:A24)							
33	Formula in cell C25	=C16*SUMPRODUCT(\$A\$18:\$A\$24,C18:C24)							
34	Formula in cell D25-I25	Copied from C25 (note the absolute addresses)							
35	Formula in cell A26	=SUMPRODUCT(\$A\$18:\$A\$24,H6:H12)							
36	Formula in cell A27	=SUM(C25:I25)^0.5							
37	Formula in cell A28	=A26/A27							
38									

	A	B	C	D	E	F	G	H	I	J	K	L	
39	7A.3 The Efficient Frontier												
40													
41	Cell to store constraint on risk premium				0.0400								
42													
43			Min Var					Optimum					
44	Mean		0.0383	0.0400	0.0450	0.0500	0.0550	0.0564	0.0575	0.0600	0.0700	0.0800	
45	SD	0.1	0.1132	0.1135	0.1168	0.1238	0.1340	0.1374	0.1401	0.1466	0.1771	0.2119	
46	Slope		0.3386	0.3525	0.3853	0.4037	0.4104	0.4107	0.4106	0.4092	0.3953	0.3774	
47	US		0.6112	0.6195	0.6446	0.6696	0.6947	0.7018	0.7073	0.7198	0.7699	0.8201	
48	UK		0.8778	0.8083	0.5992	0.3900	0.1809	0.1214	0.0758	-0.0283	-0.4465	-0.8648	
49	France		-0.2140	-0.2029	-0.1693	-0.1357	-0.1021	-0.0926	-0.0852	-0.0685	-0.0014	0.0658	
50	Germany		-0.5097	-0.4610	-0.3144	-0.1679	-0.0213	0.0205	0.0524	0.1253	0.4185	0.7117	
51	Australia		0.0695	0.0748	0.0907	0.1067	0.1226	0.1271	0.1306	0.1385	0.1704	0.2023	
52	Japan		0.2055	0.1987	0.1781	0.1575	0.1369	0.1311	0.1266	0.1164	0.0752	0.0341	
53	Canada		-0.0402	-0.0374	-0.0288	-0.0203	-0.0118	-0.0093	-0.0075	-0.0032	0.0139	0.0309	
54	CAL*	0.0411	0.0465	0.0466	0.0480	0.0509	0.0550	0.0564	0.0575	0.0602	0.0727	0.0871	
55	*Risk premium on CAL = SD × slope of optimal risky portfolio												

Spreadsheets 7A.1, 7A.2, 7A.3

Spreadsheet model for international diversification

matrix is not corrected for degrees-of-freedom bias; hence, each of the elements in the matrix was multiplied by 60/59 to eliminate downward bias.

Expected Returns

While estimation of the risk parameters (the covariance matrix) from excess returns is a simple technical matter, estimating the risk premium (the expected excess return) is a daunting task. As we discussed in Chapter 5, estimating expected returns using historical data is unreliable. Consider, for example, the negative average excess returns on U.S. large stocks over the period 2001–2005 (cell G6) and, more generally, the big differences in average returns between the 1991–2000 and 2001–2005 periods, as demonstrated in columns F and G.

In this example, we simply present the manager's forecasts of future returns as shown in column H. In Chapter 8 we will establish a framework that makes the forecasting process more explicit.

The Bordered Covariance Matrix and Portfolio Variance

The covariance matrix in Spreadsheet 7A.2 is bordered by the portfolio weights, as explained in Section 7.2 and Table 7.2. The values in cells A18–A24, to the left of the covariance matrix, will be selected by the optimization program. For now, we arbitrarily input 1.0 for the U.S. and zero for the others. Cells A16–I16, above the covariance matrix, must be set equal to the column of weights on the left, so that they will change in tandem as the column weights are changed by Excel's Solver. Cell A25 sums the column weights and is used to force the optimization program to set the sum of portfolio weights to 1.0.

Cells C25–I25, below the covariance matrix, are used to compute the portfolio variance for any set of weights that appears in the borders. Each cell accumulates the contribution to portfolio variance from the column above it. It uses the function SUMPRODUCT to accomplish this task. For example, row 33 shows the formula used to derive the value that appears in cell C25.

Finally, the short column A26–A28 below the bordered covariance matrix presents portfolio statistics computed from the bordered covariance matrix. First is the portfolio risk premium in cell A26, with formula shown in row 35, which multiplies the column of portfolio weights by the column of forecasts (H6–H12) from Spreadsheet 7A.1. Next is the portfolio standard deviation in cell A27. The variance is given by the sum of cells C25–I25 below the bordered covariance matrix. Cell A27 takes the square root of this sum to produce the standard deviation. The last statistic is the portfolio Sharpe ratio, cell A28, which is the slope of the CAL (capital allocation line) that runs through the portfolio constructed using the column weights (the value in cell A28 equals cell A26 divided by cell A27). The optimal risky portfolio is the one that maximizes the Sharpe ratio.

Using the Excel Solver

Excel's Solver is a user-friendly, but quite powerful, optimizer. It has three parts: (1) an objective function, (2) decision variables, and (3) constraints. Figure 7A.1 shows three pictures of the Solver. For the current discussion we refer to picture A.

The top panel of the Solver lets you choose a target cell for the "objective function," that is, the variable you are trying to optimize. In picture A, the target cell is A27, the portfolio standard deviation. Below the target cell, you can choose whether your objective is to maximize, minimize, or set your objective function equal to a value that you specify. Here we choose to minimize the portfolio standard deviation.

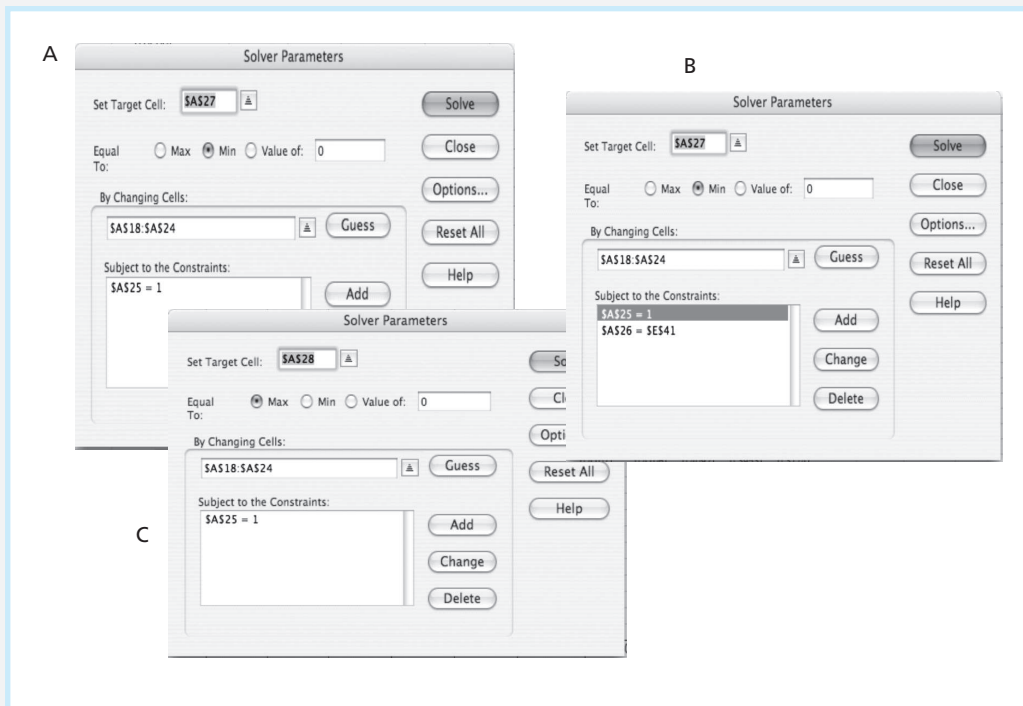


Figure 7A.1 Solver dialog box

The next panel contains the decision variables. These are cells that the Solver can change in order to optimize the objective function in the target cell. Here, we input cells A18–A24, the portfolio weights that we select to minimize portfolio volatility.

The bottom panel of the Solver can include any number of constraints. One constraint that must always appear in portfolio optimization is the “feasibility constraint,” namely, that portfolio weights sum to 1.0. When we bring up the constraint dialogue box, we specify that cell A25 (the sum of weights) be set equal to 1.0.

Finding the Minimum Variance Portfolio

It is helpful to begin by identifying the global minimum variance portfolio (*G*). This provides the starting point of the efficient part of the frontier. Once we input the target cell, the decision variable cells, and the feasibility constraint, as in picture A, we can select “solve” and the Solver returns portfolio *G*. We copy the portfolio statistics and weights to our output Spreadsheet 7A.3. Column C in Spreadsheet 7A.3 shows that the lowest standard deviation (SD) that can be achieved with our input list is 11.32%. Notice that the SD of portfolio *G* is considerably lower than even the lowest SD of the individual indexes. From the risk premium of portfolio *G* (3.83%) we begin building the efficient frontier with ever-larger risk premiums.

Charting the Efficient Frontier of Risky Portfolios

We determine the desired risk premiums (points on the efficient frontier) that we wish to use to construct the graph of the efficient frontier. It is good practice to choose more points in the neighborhood of portfolio *G* because the frontier has the greatest curvature in that region. It is sufficient to choose for the highest point the highest risk premium from the

input list (here, 8% for Germany). You can produce the entire efficient frontier in minutes following this procedure.

1. Input to the Solver a constraint that says: Cell A26 (the portfolio risk premium) must equal the value in cell E41. The Solver at this point is shown in picture B of Figure 7A.1. Cell E41 will be used to change the required risk premium and thus generate different points along the frontier.
2. For each additional point on the frontier, you input a different desired risk premium into cell E41, and ask the Solver to solve again.
3. Every time the Solver gives you a solution to the request in (2), copy the results into Spreadsheet 7A.3, which tabulates the collection of points along the efficient frontier. For the next step, change cell E41 and repeat from step 2.

Finding the Optimal Risky Portfolio on the Efficient Frontier

Now that we have an efficient frontier, we look for the portfolio with the highest Sharpe ratio. This is the efficient frontier portfolio that is tangent to the CAL. To find it, we just need to make two changes to the Solver. First, change the target cell from cell A27 to cell A28, the Sharpe ratio of the portfolio, and request that the value in this cell be maximized. Next, eliminate the constraint on the risk premium that may be left over from the last time you used the Solver. At this point the Solver looks like picture C in Figure 7A.1.

The Solver now yields the optimal risky portfolio. Copy the statistics for the optimal portfolio and its weights to your Spreadsheet 7A.3. In order to get a clean graph, place the column of the optimal portfolio in Spreadsheet 7A.3 so that the risk premiums of all portfolios in the spreadsheet are steadily increasing from the risk premium of portfolio G (3.83%) all the way up to 8%.

The efficient frontier is graphed using the data in cells C45–I45 (the horizontal or *x*-axis is portfolio standard deviation) and C44–I44 (the vertical or *y*-axis is portfolio risk premium). The resulting graph appears in Figure 7A.2.

The Optimal CAL

It is instructive to superimpose on the graph of the efficient frontier in Figure 7A.2 the CAL that identifies the optimal risky portfolio. This CAL has a slope equal to the Sharpe ratio of

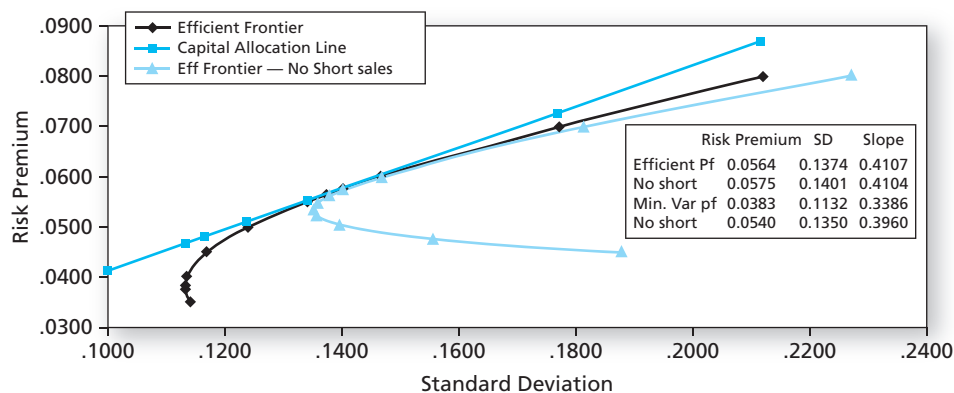


Figure 7A.2 Efficient frontier and CAL for country stock indexes

the optimal risky portfolio. Therefore, we add at the bottom of Spreadsheet 7A.3 a row with entries obtained by multiplying the SD of each column's portfolio by the Sharpe ratio of the optimal risky portfolio from cell H46. This results in the risk premium for each portfolio along the CAL efficient frontier. We now add a series to the graph with the standard deviations in B45–I45 as the x -axis and cells B54–I54 as the y -axis. You can see this CAL in Figure 7A.2.

The Optimal Risky Portfolio and the Short-Sales Constraint

With the input list used by the portfolio manager, the optimal risky portfolio calls for significant short positions in the stocks of France and Canada (see column H of Spreadsheet 7A.3). In many cases the portfolio manager is prohibited from taking short positions. If so, we need to amend the program to preclude short sales.

To accomplish this task, we repeat the exercise, but with one change. We add to the Solver the following constraint: Each element in the column of portfolio weights, A18–A24, must be greater than or equal to zero. You should try to produce the short-sale constrained efficient frontier in your own spreadsheet. The graph of the constrained frontier is also shown in Figure 7A.2.



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