HW 1: due Friday Week 2

Q1)

Download 'Group Velocity' program from the class website and adjust the dispersion relation, k(ω) so that
(i) Vg = Vp
(ii) Vg < Vp
(iii) Vg > Vp
You can check your answer by running the animation yourself. Unfortunately, we can see it from your printout.

Q2) Vector Calculus:

a) Two vectors in Cartesian coordinates are given as

$$\vec{A} = (2 + j3)\hat{x} + (3 - j)\hat{y}, \vec{B} = (1 + j)\hat{x} + \frac{j}{2}\hat{y}$$

Calculate $|\vec{A}|$, $|\vec{B}|$, $\vec{A} \cdot \vec{B}$, and $\vec{A} \times \vec{B}$.

b) Given the vector field $\vec{A} = (x - jy)\hat{x} + y^2\hat{z}$, calculate $\nabla . \vec{A}$, $\nabla \times \vec{A}$, and $\nabla . (\nabla \times \vec{A})$.

Q3) Maxwell's Wave Equation:
Consider travelling sinusoidal wave of the form $E = \frac{E_0}{r}\cos(\omega t - kr + \varphi_0)$. Show that this scalar field representation of spherical wave satisfies Maxwell's wave equation in free space (or air) that is in the form of

$$\nabla^2 E - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0$$

Q4) Refractive Index and Dispersion: Use Sellmeier equation to assess the dispersion relation of pure silica glass (SiO_2) that is the main material of optical fibers. Plot refractive index (n), group index (N_g), material dispersion $\left(D_M = \frac{1}{c}\frac{dn_g}{d\lambda}\right)$ and dispersion slope parameter $\left(S = \frac{dD}{d\lambda}\right)$ in the range of 500nm-1900nm. Report the values at 1550nm. Include your Mathematica script with your results.