Build a Hidden Markov Model for the following (unrealistically simplified) problem of detecting a sensor failure in a sensor network:

In an intelligent environment we have a magnetic Hall-effect sensor that detects whether a door is opened or closed. The output of the sensor has 3 possible states, "O, C, U", representing open, closed, and unknown, respectively (the unknown arises due to fluctuations in the magnetic field

of the sensor). Given the sensor characteristics, the noise level of the working sensor is such that it detects the correct state of the door 75% of the time and is in an unknown state the other 25% of the time (there are no false door state detections).

The sensor node measures the state of the door (open or closed) once every 15 seconds and transmits it to a central computer for processing.

Due to a bug in the data acquisition part of the sensor node, the data acquisition unit which reads the actual sensor signal from time to time freezes and if it does so, the sensor node strictly returns the last correct door state reading (i.e. is stuck on open or closed). Note that the data transmission is not affected and thus the sensor node will continue to transmit this data. When the data acquisition is frozen, random fluctuations in the wiring can cause a reset which will "unfreeze" it and return it to normal operation. Measurements have shown that on average, the data acquisition unit freezes once every 4 minutes and stays in the frozen state on average for 1 minute.

a) Build a Hidden Markov Model that represents the scenario described above (you can make any assumption about the opening and closing of the door, including that it whether it is open or closed can change randomly at any point in time) that at the beginning the data acquisition unit operates correctly.

b) Use this model and the Viterbi algorithm to evaluate at what times most likely the data acquisition unit was "frozen" and the transmitted data should therefore not be trusted for the following observation sequence:

$$CCCUCOUCCUCCCCCCCCCUOOCUOOCUOO$$

Determine the most likely state sequence, and indicate which of the readings should be ignored (since they correspond to failure states). Also list the intermediate values (δ and Ψ) of the Viterbi (again, submit these tables electronically as a text file).