5.1 Random Variables

Randomness plays an important part in our lives. Most things happen randomly. For example, consider the following events: winning a lottery, your car breaking down, getting sick, getting involved in an accident, losing a job, making money in the stock market, and so on. We cannot predict when, where, and to whom these things can or will happen. Who will win a lottery and when? Who will get sick and when? Where, when, and who will get involved in an accident? All these events are uncertain, and they happen randomly to people. In other words, there is no definite time, place, and person for these events to happen. Similarly, consider the following events: how many customers will visit a bank, a grocery store, or a gas station on a given day? How many cars will pass a bridge on a given day? How many students will be absent from a class on a given day? In all these examples, the *number* of customers, cars, and students are random; that is, each of these can assume any value within a certain interval.

Suppose Table 5.1 gives the frequency and relative frequency distributions of the number of vehicles owned by all 2000 families living in a small town.

Table 5.1 Frequency and Relative Frequency Distributions of the Number of Vehicles Owned by Families

Number of Vehicles Owned	Frequency	Relative Frequency
0	30	30/2000 = .015
1	320	320/2000 = .160
2	910	910/2000 = .455
3	580	580/2000 = .290
4	160	160/2000 = .080
	N = 2000	Sum = 1.000

Suppose one family is randomly selected from this population. The process of randomly selecting a family is called a *random* or *chance experiment*. Let x denote the number of vehicles owned by the selected family. Then x can assume any of the five possible values (0, 1, 2, 3, and 4) listed in the first column of Table 5.1. The value assumed by x depends on which family is selected. Thus, this value depends on the outcome of a random experiment. Consequently, x is called a **random variable** or a **chance variable**. In general, a random variable is denoted by x or y .

Random Variable A **random variable** is a variable whose value is determined by the outcome of a random experiment.

As will be explained next, a random variable can be discrete or continuous.

5.1.1 Discrete Random Variable

A **discrete random variable** assumes values that can be counted. In other words, the consecutive values of a discrete random variable are separated by a certain gap.

Discrete Random Variable A random variable that assumes countable values is called a **discrete random variable.**

In Table 5.1, *the number of vehicles owned by a family* is an example of a discrete random variable because the values of the random variable x are countable: 0, 1, 2, 3, and 4. Here are a few other examples of discrete random variables:

1. The number of cars sold at a dealership during a given month
2. The number of houses in a certain block

3.

The number of fish caught on a fishing trip

4. The number of complaints received at the office of an airline on a given day
5. The number of customers who visit a bank during any given hour
6. The number of heads obtained in three tosses of a coin

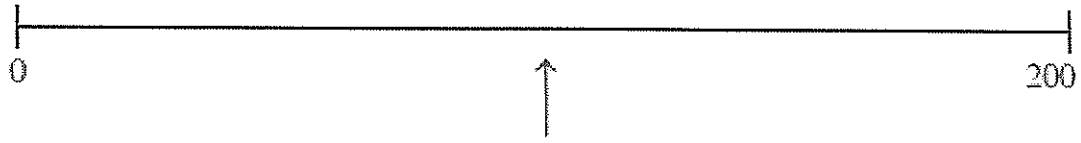
5.1.2 Continuous Random Variable

A random variable whose values are not countable is called a **continuous random variable.**

A continuous random variable can assume any value over an interval or intervals.

Continuous Random Variable A random variable that can assume any value contained in one or more intervals is called a **continuous random variable.**

Because the number of values contained in any interval is infinite, the possible number of values that a continuous random variable can assume is also infinite. Moreover, we cannot count these values. Consider the life of a battery. We can measure it as precisely as we want. For instance, the life of this battery may be 40 hours, or 40.25 hours, or 40.247 hours. Assume that the maximum life of a battery is 200 hours. Let x denote the life of a randomly selected battery of this kind. Then, x can assume any value in the interval 0 to 200. Consequently, x is a continuous random variable. As shown in the diagram below, every point on the line representing the interval 0 to 200 gives a possible value of x .



Every point on this line represents a possible value of x that denotes the life of a battery. There is an infinite number of points on this line. The values represented by points on this line are uncountable.

The following are a few other examples of continuous random variables:

1. The length of a room
2. The time taken to commute from home to work
3. The amount of milk in a gallon (note that we do not expect "a gallon" to contain exactly one gallon of milk but either slightly more or slightly less than one gallon).
4. The weight of a letter
5. The price of a house

Note that money is often treated as a continuous random variable, specifically when there are a large number of unique values.

This chapter is limited to the discussion of discrete random variables and their probability distributions. Continuous random variables will be discussed in Chapter 6.