Lecture 5

The T-Test

In the previous lecture, we introduced the hypothesis testing procedure, and developed the first two steps of a statistical test to determine if male and female mean salaries could be equal in the population – where our differences were caused simply by sampling errors. This lecture continues with this example by completing the final three steps. It also introduces our first statistical test, the t-test for mean equality.

Last week we looked at the normal curve and noted several of its characteristics, such as mean = median = mode, symmetrical around the mean, curve height drops off the further the score gets from the mean (meaning scores further from the mean are less likely to occur). Our first statistical test, the *t-test*, is based on a population that is distributed normally. The t-test is used when we do not have the population variance value – this is the situation every time we use a sample to make decisions about their related populations.

While the t-test has several different versions, we will focus on the most commonly used form – the *two sample test for mean equality assuming equal variance*. When we are testing measures for mean equality, it is fairly rare for the variances to be much difference, and the observed difference is often merely sample error. (In Lecture 6, we will revisit this assumption.)

The logic of the test is that the difference between mean values divided by a measure of this difference's variation will provide a t statistic that is distributed normally, with the mean equaling 0 and the standard deviation equaling 1. This outcome can then be tested to see what the likelihood is that we would get a value this large or larger purely by chance – our old friend the *p-value*. If this p-value exceeds our decision criteria, *alpha*, then we reject the null hypothesis claim of no difference (Lind, Marchel, & Wathen, 2008).

Setting up the t-test

Before selecting any test from Excel, the data needs to be set up. For the t-test, there are a couple of steps needed. First, copy the data you want to first set up the data. In our question about male and female salaries, copy the *gender* variable column from the data page to a new worksheet page (the recommendation is on the week 2 tab) and paste it to the right of the questions (such as in column T), then copy and paste the *salary* values and paste them next to the gender data. Next, sort both columns by the gender column – this will give you the salary data sorted by gender. Then, in column V place the label/word Males, and in column W place the label Females. Now copy the male salaries and paste them under the Male label, and do the same for the female salaries and the female label. The data is now set up for easy entry into the T-test data entry section.

The t-test is found in the Analysis Toolpak that was loaded into your Excel program last week. To find it, click on the Data button in the top ribbon, then on the Data Analysis link in the Analyze box at the right, then scroll down to the T-test: Two-Sample Assuming Equal Variances. For assistance in setting up the t-test, please see the discussion in the Week 2 Excel Help lecture.

Interpreting the T-test Output

The t-test output contains a lot of information, and not all of it is needed to interpret the result. The important elements of the t-test outcome will be shown with an example for our research case question.

Equal Pay Example - continued

In Lecture 4 we set up the first couple of steps for our testing of the research question: Do males and females receive equal pay for equal work? Our first examination of the data we have for answering this question involves determining if the average salaries are the same.

Here is the completed hypothesis test for the question: Is the male average salary equal to the female average salary?

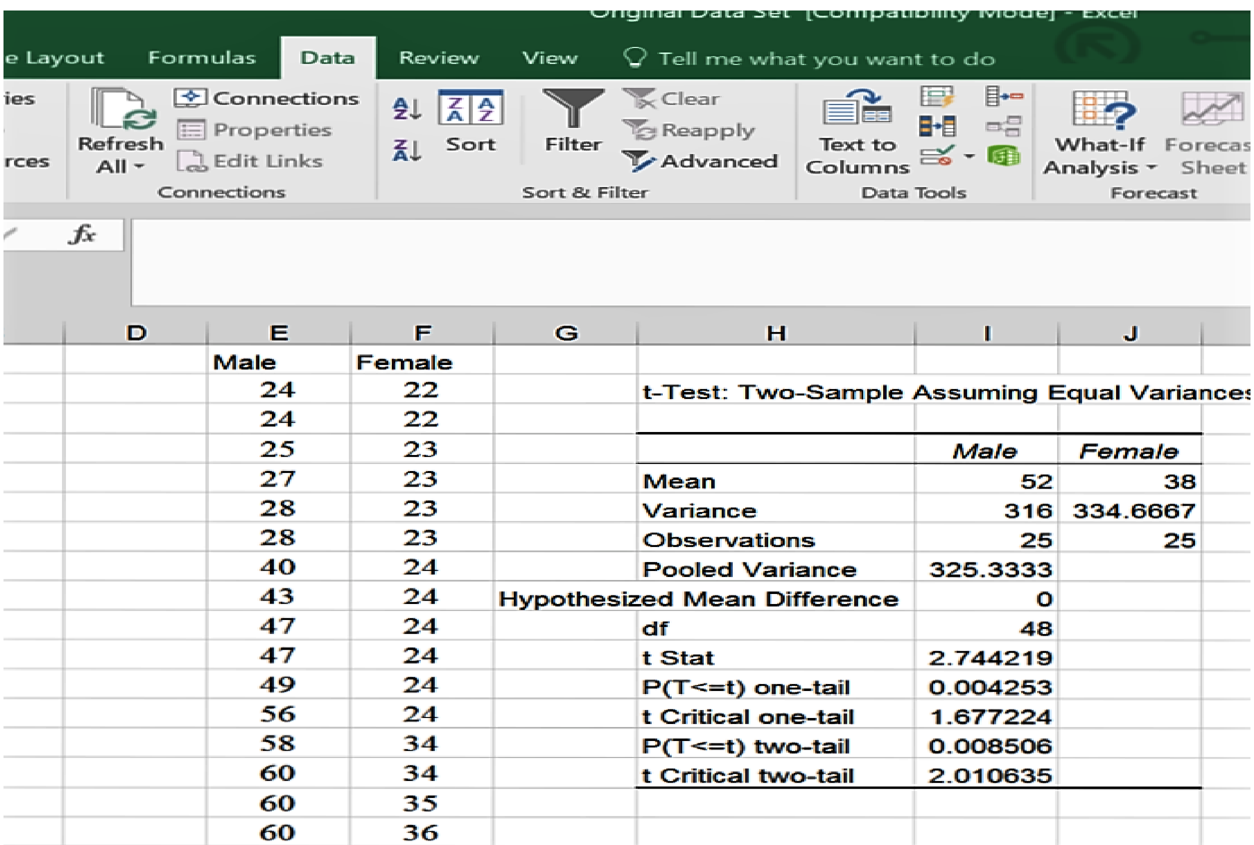
Step 1. Ho: Male mean salary = female mean salary

Ha: Male mean salary ≠ female mean salary

Step 2. Reject the null if the p-value is < (less than) alpha = .05.

Step 3. The selected test is the Two-Sample T-test assuming equal variances.

Step 4. The test results are below. The screen shot shows output table.



Male	Female
24	22
24	22
25	23
27	23
28	23
28	23
40	24
43	24
47	24
47	24
49	24
56	24
58	34
60	34
60	35
60	36

t-Test: Two-Sample Assuming Equal Variances

	Male	*Female*
Mean	52	38
Variance	316	334.6667
Observations	25	25
Pooled Variance	325.3333	
Hypothesized Mean Difference	0	
df	48	
t Stat	2.744219	
P(T<=t) one-tail	0.004253	
t Critical one-tail	1.677224	
P(T<=t) two-tail	0.008506	
t Critical two-tail	2.010635	

Step 5. Interpretation and conclusions.

The first step is to ensure we have all of the correct data. We see that we have 25 males and females in the Observations row, and that the respective means are equal to what we earlier calculated.

The calculated t statistic is 2.74 (rounded). We have two ways to determine if our result rejects or fails to reject the null hypothesis; both involve the two-tail rows, as we have a two tail test (equal or not equal hypothesis statements). The first is a comparison of the t-values – if the critical t of 2.74 (rounded) is greater than the T-Critical two-tail value of 2.01, we reject the null hypothesis. The second way is to compare the p-value with our criteria of alpha = .05. Remember, since this is a two-tail test, the alpha for each tail is half of the overall alpha or .025. If the p-value (shown as P(T<=t) two -tail value of 0.0085 is less than our one tail alpha (.025) then we reject the null hypothesis. Note: at times Excel will report the p-value in an E format, such as 3.45E-04. This is called an Exponent format, and is the same as $3.45 * 10^{-04}$. This means move the decimal point 4 places to the left, making 3.45E-04 = 0.000345. Virtually any p-value reported with an E-xx form will be less than our alpha of 0.05 (which would be 5E-02).

Since we rejected the null hypothesis in both approaches (and both will always provide the same outcome), we can answer our question with: No - the male and female mean salaries are not equal.

Note that for this set of data, we would have rejected the null for a one-tail test if and only if the null hypothesis had been: Male mean salary is <= Female mean salary and the alternate was Male mean salary is > Female mean salary. The arrow in the alternate points to the positive/right tail and that is where the calculated t-statistic is. So, even if the p-value is smaller than alpha in a one tail test, we need to ensure the t-statistic is in the correct tail for rejection.

References

Lind, D. A., Marchel, W. G., & Wathen, S. A. (2008). *Statistical Techniques in Business & Finance*. (13th Ed.) Boston: McGraw-Hill Irwin.