1. Determine if each of the Hessians below are positive definite (PD), positive semi-definite (PSD), or something else, using superdiagonalization. Clearly indicate how you know your conclusion is correct. You may use MS Excel to do individual calculations, but if you do, include the Excel file with your submission.
$\boldsymbol{H}_{a}(\overrightarrow{\boldsymbol{x}})=\left[\begin{array}{cccc}25 & 10 & 5 & 30 \\ 10 & -5 & -1 & 10 \\ 5 & -1 & 26 & 4 \\ 30 & 10 & 4 & 53\end{array}\right]$
$\boldsymbol{H}_{b}(\overrightarrow{\boldsymbol{x}})=\left[\begin{array}{cccc}15 & 8 & 3 & 10 \\ 8 & 12 & -4 & 0 \\ 3 & -4 & 0 & 4 \\ 10 & 0 & 4 & 21\end{array}\right]$
$\boldsymbol{H}_{c}(\overrightarrow{\boldsymbol{x}})=\left[\begin{array}{cccc}4 & 6 & 3 & 14 \\ 6 & 18 & 22 & 20 \\ 3 & 22 & 52 & 10 \\ 14 & 20 & 10 & 58\end{array}\right]$
2. Consider the following function. For maximum credit, answer the following questions using the theory discussed so far in the lectures and the assigned reading, without using a calculator or a computer. Be sure to identify the reason for each conclusion and clearly state your conclusion. Also, you may not use a plot to answer any of the questions.

$$
f(x)=x^{4}-3 x^{2}+5
$$

a. Find all the critical points of $f(x)$ and determine the nature of each critical point.
b. On the open interval $(0,2)$, is $f(x)$ any one of convex, quasiconvex, or pseudoconvex?
3. For maximum credit, show all steps and clearly indicate your answer. Also, do not use any calculator, computer, or plotter to solve this problem.
Consider the following function.

$$
f(\overrightarrow{\boldsymbol{x}})=\left(x_{1}^{2}-2 x_{2}-3\right)^{2}+\left(x_{2}-2 x_{3}\right)^{2}
$$

a. First, find $\nabla f(\overrightarrow{\boldsymbol{x}})$ and identify all critical points of the function.
b. Then, find $\boldsymbol{H}(\overrightarrow{\boldsymbol{x}})$ and then tell me the nature of each point and how you know you are correct.
4. The following vectors form a basis. Use the Gram-Schmidt procedure to create an orthogonal unit-length set of vectors from this basis. In your answer, show how you determined each $\overrightarrow{\boldsymbol{b}}_{j}$ and $\overrightarrow{\boldsymbol{d}}_{j}^{\text {new }}$. You can use a computer program to do your calculations, but not pre-written Gram-Schmidt code. Include your code with your submission.

$$
\overrightarrow{\boldsymbol{a}}_{1}=\left[\begin{array}{c}
5 \\
-2 \\
6
\end{array}\right], \quad \overrightarrow{\boldsymbol{a}}_{2}=\left[\begin{array}{c}
7 \\
3 \\
-1
\end{array}\right] \quad \text { and } \quad \overrightarrow{\boldsymbol{a}}_{3}=\left[\begin{array}{c}
-2 \\
-4 \\
3
\end{array}\right]
$$

5. Your boss has used MS Excel Solver to find a solution that minimizes the following function. Note that x is not constrained.

$$
f(x)=0.75 x^{4}-7 x^{3}+10.5 x^{2}+45 x+36
$$

He insists that the optimal solution is at 5.00 because Solver found that result when he started at $x=4.00$. At $x=5 . f(x)=81.25$.
a. Without using a computer or calculator, what do you think about the answer your boss got?
b. What do you say to your boss? For this part, you may use MS Excel, including Solver and plots the function when preparing your answer to your boss.

Instructions:

- You may use your text, your notes, and a standard calculator.
- You may not use a computer for a question, unless the problem says so. Whenever you use a computer, include any computer files with your submission.
- For maximum credit, clearly state your answer on a version of this test, show all necessary work.
- You can enter answers and work by editing a copy of this test, or
- Print on this test, write on it, then scan ( 300 dpi ) and submit the scan.
- In either case you can attach any calculations to the test (edited or scanned).
- Don't forget this page and any computer files.

