Mini-Application Project
Equilibrium Temperature Distribution

Overview

A common problem encountered in thermodynamics is that of solving for the equilibrium temperature distribution of a thin plate of metal. One way of solving this type of problem is to solve a continuous-time differential equation that can be queried as a function of *(x,y)* for any continuous-valued position on the plate of metal. In general, this solution could be exact given certain assumptions, but this solution is somewhat difficult to compute. A simpler way to approximately solve the problem is to discretize the plate and solve a system of linear equations (for example, see the related topic of "finite element methods"). The solution will be found by solving this system of linear equations using the matrix inverse.

Problems

Consider the discretized square plate in Figure 1.

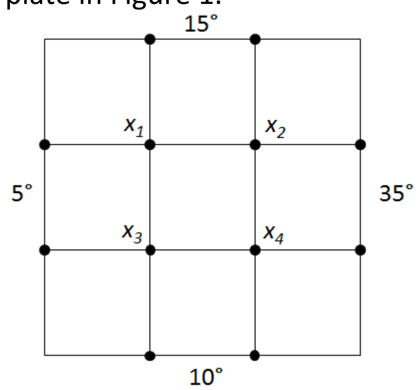

Figure 1 – Discretized Square Plate

1. Write a system of linear equations for this system and clearly identify the matrix A in the matrix equation $A\boldsymbol{x}=\boldsymbol{b}$ by averaging the values at each node x_1, x_2, x_3, and x_4 by taking the (thermal) average of its (four) adjacent nodes (with the given boundary conditions).

2. Compute the inverse of A using an augmented matrix and row operations.

3. Find the solution of the system of equations by solving the matrix equation $A\boldsymbol{x}=\boldsymbol{b}$ by multiplying both sides (on the left) by the inverse of A.

4. If the given nodes were part of a larger 100 by 100 grid (with this square in the upper left corner), what would the matrix A look like (draw a representation of this matrix indicating zero values, etc.)? (Such matrices are called "sparse".)