## Homework Set \#4

1. ( $\mathbf{2 4} \mathbf{~ p t s})$ A bandlimited continuous-time signal is known to contain 60 Hz component which we want to remove by processing with the system shown below. Assume that $T=10^{-4}$.

(a) ( $6 \mathbf{p t s}$ ) What is the highest frequency that the continuous time signal can contain if aliasing is to be avoided?
(b) ( $\mathbf{1 2} \mathbf{~ p t s ) ~ T h e ~ d i s c r e t e - t i m e ~ s y s t e m ~ t o ~ b e ~ u s e d ~ h a s ~ t h e ~ f o l l o w i n g ~ f r e q u e n c y ~ r e s p o n s e : ~}$

$$
H\left(e^{j \omega}\right)=\frac{\left[1-e^{-j\left(\omega-\omega_{0}\right)}\right]\left[1-e^{-j\left(\omega+\omega_{0}\right)}\right]}{\left[1-0.9 e^{-j\left(\omega-\omega_{0}\right)}\right]\left[1-0.9 e^{-j\left(\omega+\omega_{0}\right)}\right]}
$$

Using Matlab, plot the magnitude and phase of $H\left(e^{j \omega}\right)$ for $\omega_{0}=\pi / 2$. Please submit your code and plots.
(c) ( $\mathbf{6} \mathbf{p t s}$ ) Which value of $\omega_{0}$ would you choose to eliminate the 60 Hz component?
2. ( $\mathbf{1 6} \mathbf{p t s}$ ) Consider the system in the figure below with $X_{c}(j \Omega)=0$ for $|\Omega| \geq 2 \pi(1000)$ and the discrete time system a squarer, i.e. $y[n]=x^{2}[n]$. What is the largest value of $T$ such that $y_{c}(t)=x^{2}(t)$ ?

3. ( $\mathbf{1 6} \mathbf{p t s}$ ) Assume that in figure below $X_{c}(j \Omega)=0$ for $|\Omega| \geq \pi / T_{1}$. For the general case of $T_{1} \neq T_{2}$, express $y_{c}(t)$ in terms of $x_{c}(t)$.

4. $(20 \mathrm{pts})$ In the figure below, $x[n]=x_{c}(n T)$ and $y[n]=x[2 n]$.

(a) Assume that $x_{c}(t)$ has a Fourier transform such that $X_{c}(j \Omega)=0,|\Omega|>2 \pi(100)$. What value of $T$ is required so that

$$
X\left(e^{j \omega}\right)=0, \quad \frac{\pi}{2}<|\omega| \leq \pi ?
$$

(b) How should $T^{\prime}$ be chosen so that $y_{c}(t)=x_{c}(t)$ ?
5. (24 pts) Consider the system shown below. For each of the following input signals $x[n]$, indicate whether the output $x_{r}[n]=x[n]$.

(a) (8 pts) $x[n]=\cos (\pi n / 4)$
(b) $(8 \mathrm{pts}) x[n]=\cos (\pi n / 2)$
(c) $(8 \mathrm{pts})$

$$
x[n]=\left[\frac{\sin (\pi n / 8)}{\pi n}\right]^{2}
$$

Hint: Use the modulation property of the Fourier transform to find $X\left(e^{j \omega}\right)$.

