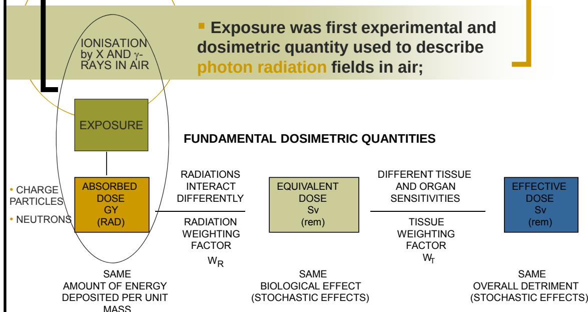
Suggested reading: Cember, Chapters 6 and 10

External Radiation Dosimetry of Photons

- Exposure – Dose relationship
- Gamma Radiation Dosimetry
  - Basic Source Geometries:
    - Point source
    - Line source
    - Plain (Disk) source
    - Slab source
- Kerma and Dose

Review: Radiation Dosimetry

- **Exposure was first experimental and dosimetric quantity used to describe photon radiation fields in air;**



Review: Interactions of photons with matter

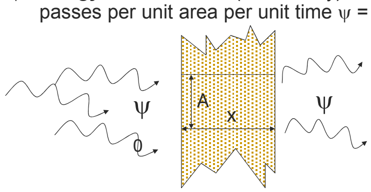
- Principal processes:
  - Photoelectric effect (interaction photon-atom)
  - Compton scattering (inelastic scattering)
  - Pair production
- Minor processes:
  - Photo-nuclear reactions
  - Elastic scattering (Rayleigh and Thompson)

After an interaction, a photon either disappears or is scattered.

As a result of these interactions energy is deposited in a medium and secondary charges are produced, which are also depositing part or all of their energy by ionising and exciting the atoms of medium.

Review of terms

- Φ: **Fluence** is the number of particles per unit area that cross a plain perpendicular to the beam; unit: γ m^{-2}
- ϕ: **Flux or fluence rate** is a number of particles crossing unit area per unit time; unit: γ $m^{-2}s^{-1}$
- Ψ: **Energy Fluence** is energy that passes per unit area $\Psi = \Phi \cdot E$
- ψ: **Energy Fluence Rate** (or intensity) is energy that passes per unit area per unit time $\psi = \phi \cdot E$



Exposure

- First dosimetric quantity for **photon radiation** fields;
- Based upon the ionization of air produced by the field;
- **The exposure** is the amount of charge of ions (of either sign) produced per unit mass of air under conditions of **charged particle equilibrium (see slide #37).**
- The charge involved is the total charge (i.e. primary, secondary etc.) of one sign;
- Unit: **1 R = 2.58 x 10^{-4} C/kg**

Exposure in air

- Determine the number of ion-pairs per a Roentgen.

1 R = 2.58 x 10^{-4} C/kg / 1.6 x 10^{-19} C/ion pair

- How much energy is used to produce a ion-pair?

Remember in air W = 34 eV per ion pair, so

and going back to a Roentgen

$$1\,\mathrm{R} = \frac{2.58 \times 10^{-4}\ \mathrm{C/kg}}{1.6 \times 10^{-19}\ \mathrm{C/ion\ pair}} 34eV \times \frac{1.6 \times 10^{-19}\ \mathrm{J}}{1eV}$$

$$1\,\mathrm{R} = 0.0088 \frac{\mathrm{J}}{\mathrm{kg}} = 8.8\mathrm{cGy}$$

This is a dose of 1 R to air, since w_R = **1** for x- and γ-rays!

This is also a unit conversion from exposure to a dose in air!

Exposure (cont'd)

- **The exposure** may be related to the energy deposited in the mass of air, since on the average **W = 34 eV** is required to produce an ion pair contributing with e = 1.6×10^{-19} C of charge in the air.
- Exposure is an integrated measure and it is independent of the time of which the exposure occurs.
- The strength of an X-ray or gamma field can be expressed as an exposure rate,

$$\frac{dX}{dt} = \dot{X}\,(=)\frac{C}{kg \cdot s}$$

- **The total exposure** is thus the product of exposure rate and time during which exposure occurred:

$$X = \dot{X} \cdot t\,(=)\frac{C}{kg}$$

Exposure to dose conversions

- The primary result of the interaction of a radiation field with matter is the transfer of energy, which may then generally result in ionization or excitation of the medium.
- The dose is defined as the energy of ionization and excitation absorbed per unit mass:

$$D = \frac{E}{m}$$

- The ionization produced by a photon radiation field results from the transfer of energy, thus that **the exposure is directly related to the dose to air.**
- For W=34 eV of energy transfer charge e is produced, so the total charge per unit mass may be written

$$X = \frac{e}{W} D_{air}$$

Source Strength: Specific Γ Constant or Specific γ- ray Emission of a Point Source of γ-rays

- **Specific γ- ray Emission gives the exposure rate per unit activity at unit distance (1 m) from a point source**
  - In standard units, R m^2 $Ci^{-1}$$h^{-1}$
  - Symbol Γ or **G**
  - *Note – symbols can change – concepts stay the same!!!*

Source Strength: Specific Γ Constant or Specific γ- ray Emission of a Point Source of γ-rays

- If S(E) is the source strength (activity) of a point source which emits photons of energy E, the fluece rate will be

$$\dot{\Phi} = \frac{S(E)}{4\pi r^2}$$

but a gamma **source strength is more generally the product of activity and probability of emission** (branching ration), thus

$$\dot{\Phi} = \frac{p(E)N\lambda}{4\pi r^2}$$

Source Strength: Specific Γ Constant or Specific γ- ray Emission of a Point Source of γ-rays

- The energy rate is

$$\dot{\psi} = E \cdot \dot{\Phi} = E\frac{p(E)N\lambda}{4\pi r^2}$$

- Then the rate of energy absorbed in air is

$$\dot{D} = \dot{\psi}(E,r) \cdot (\frac{\mu_{en}}{\rho})air = \frac{E \cdot p(E)N\lambda}{4\pi r^2}(\frac{\mu_{en}}{\rho})air$$

Source Strength: Specific Γ Constant or Specific γ- ray Emission of a Point Source of γ-rays

- To calculate the exposure rate in air we will use the average energy (W = 34 eV / ion pair) required to produce an ion-pair in air.
- So the exposure rate of a mono-energetic source is

$$\dot{X} = \frac{e}{W_{air}}\dot{D} = \frac{e}{W_{air}}\frac{E \cdot p(E)N\lambda}{4\pi r^2}(\frac{\mu_{en}}{\rho})air$$

- **If a γ source emits more than one energy, the exposure rate becomes**

$$\dot{X} = \frac{e}{W_{air}}\frac{N\lambda}{4\pi r^2}\sum_{i=1}^{n} E_i \cdot p_i(E)(\frac{\mu_{en}}{\rho})_i\, air$$

Source Strength: Specific Γ Constant or Specific γ- ray Emission of a Point Source of γ-rays

- The exposure rate of γ source is

$$\dot{X} = \frac{e}{W_{air}} \frac{N\lambda}{4\pi r^2} \sum_{i=1}^{n} E_i \cdot p_i(E)\left(\frac{\mu_{en}}{\rho}\right)_i air$$

- **The specific γ-ray emission is defined as**

$$\Gamma = \frac{e}{4\pi \cdot W_{air}} \sum_{i=1}^{n} E_i \cdot p_i(E)\left(\frac{\mu_{en}}{\rho}\right)_i air$$

- And the exposure rate from a point gamma radiation source becomes

$$\dot{X} = \Gamma \frac{N\lambda}{r^2} = \Gamma \frac{A}{r^2}$$

Exposure rate from a point γ radiation source

$$\dot{X}_o(r) = \Gamma \frac{A}{r^2}$$

where

- $X_o(r)$ is the unattenuated exposure rate (R/h)
- A is a source activity (Ci)
- Γ is a specific gamma ray constant of a source (R-m²/ h-Ci)
- r is a distance from the source (m)

• This approach neglects attenuation of γ-rays in air;

- Point source approximation valid when calculation is for a distance at least three times larger than the source dimension

For a point γ-ray source…

- The unattenuated exposure rate (R/h)

$$\dot{X}_o(r) = \Gamma \frac{A}{r^2}$$

- Absorbed Dose rate (Gy/h or rad/h)

$$\dot{D}_o(r) = \frac{dE}{dm} = \frac{W}{e}\dot{X}$$

Do not forget to multiply with a exposure-to-dose conversion factor of 8.8cGy

- Equivalent Dose rate (Gy/h or rem/h)

WR = 1

$$\dot{H}_o(r) = \dot{D} w_r = \Gamma \frac{A}{r^2}$$

Source Strength: Specific Gamma-Ray Constant

- Specific γ-ray constants can be calculated using the properties of nuclides;
- Specific γ-ray constants are also tabulated;
- It can be estimated (within 20% of actual value) by (textbook page 186 equation (6.18)

$$\Gamma = 0.5\sum_i E_i\, f_i (=)\mathrm{R\ m^2\ /\ h\ Ci}$$

It is based on treating a mass energy attenuation coefficient as a constant over energy range of interest.

Find a Specific Gamma-Ray Constant for ^{24}Na.

- Information

E [MeV/decay]	p(E) or f_i	$(\mu_{en}(E)/\rho)$air [m^2/kg]
1.37	1.0	0.00254
2.75	1.0	0.00205

- For ^{24}Na it is

$$\Gamma = \frac{e}{4\pi \cdot W_{air}} \sum_{i=1}^{n} E_i \cdot p_i(E)\left(\frac{\mu_{en}}{\rho}\right)_i air$$

$$\Gamma = \frac{1.6\times10^{-19}C}{4\pi\cdot(34eV/ion-pair\times(1.6\times10^{-19}J/eV))}[1.0\times1.37\times0.00254+1.0\times2.75\times0.00205]\times10^{6}\times1.6\times10^{-19}Cm^2/kg\cdot decay=$$

$$\Gamma = 3.4\times10^{-18}\,\mathrm{C\,m^2/kg\,Bq\,s} = 1.76\,\mathrm{R\,m^2/Ci\,h}$$

- If we use the approximated formula

$$\Gamma = 0.5\sum_i E_i\, f_i\ [\mathrm{R\ m^2\ /\ Ci\ h}]$$

$$\Gamma = 0.5[1.0\times1.37+1.0\times2.75] = 2.06\ \mathrm{R\ m^2/Ci\ h}$$

Find the exposure rate and absorbed dose of a 5 mCi ^{24}Na sources at 0.3 m distance in air.

- If we assume it is a point source, the exposure rate is

$$\dot{X}\ (r) = \dot{\Gamma}\frac{A}{r^2} = (1.76\ \mathrm{R\ m^2\ /\ Ci\ h})\frac{(5\times10^{-3}\ \mathrm{Ci})}{(0.3m)^2}$$

$$\dot{X}\ (r) = 0.098\ \mathrm{R\ h^{-1}}$$

- The absorbed dose in air is

$$\dot{D}(r) = (0.098\ \mathrm{R\ h^{-1}})\left(\frac{0.0088Gy}{1R}\right)$$

$$\dot{D}\ (r) = (0.00086\ \mathrm{Gy\ h^{-1}})\ \text{at 0.3 m distance}$$

What about dose to any other material?

- On the slide # 10, we defines the rate of energy absorbed **in air** as

$$\dot{D} = \dot{\psi}(E,r)\cdot(\frac{\mu_{en}}{\rho})air = \frac{E\cdot p(E)N\lambda}{4\pi r^2}(\frac{\mu_{en}}{\rho})air$$

- The rate of **energy absorbed in a material** can be calculates using similar equation

$$\dot{D} = \dot{\psi}(E,r)\cdot(\frac{\mu_{en}}{\rho})materila = \dot{\Phi}E(\frac{\mu_{en}}{\rho})material$$

What about dose to any other material?

- Find **the absorbed dose in a human muscle,** knowing that the mass energy attenuation coefficient for muscle is 3.17 x 10^{-3} m^2/kg, from a ^{24}Na source emitting a fluence rate of 10^7 m^{-2} s^{-1} of 0.3 MeV photons incident on a muscle.

$$\dot{D} = \dot{\Phi}E(\frac{\mu_{en}}{\rho})musce = (10^7 m^{-2}s^{-1})(0.3\times10^6 eV\times\frac{1.6\times10^{-19}J}{1eV})(3.17\times10^{-3}m^2/kg)$$

$$\dot{D} = 1.52\times10^{-9}Gy/s$$

What about dose to any other material?

- Alternatively, the absorbed dose rate in a material can be calculated as the ratio of the dose rates to the material and air for a mono-energetic γ field of energy E $_{\gamma}$

$$\frac{\dot{D}m}{\dot{D}air} = \frac{\dot{\psi}(E,r)\cdot(\frac{\mu_{en}}{\rho})m}{\dot{\psi}(E,r)\cdot(\frac{\mu_{en}}{\rho})air} = \frac{(\frac{\mu_{en}}{\rho})m}{(\frac{\mu_{en}}{\rho})air}$$

- The absorbed dose in the material is

$$Dm = D\ air\ \frac{(\frac{\mu_{en}}{\rho})m}{(\frac{\mu_{en}}{\rho})air}$$

Find the absorbed dose rate of a 5 mCi ^{24}Na sources at 0.3 m distance in a muscle.

- The absorbed dose in air is

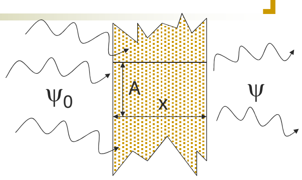
$$\dot{D}\ (r) = (0.00086\ \text{Gy h}^{-1})\ \text{at 0.3 m distance}$$

- The absorbed dose rate in a muscle is

$$\dot{D}m = \dot{D}air\frac{(\frac{\mu_{en}}{\rho})m}{(\frac{\mu_{en}}{\rho})air} = (0.00086 Gy/h)\frac{(3.17\times 10^{-3} m^2/kg)muscle}{(3.2\times 10^{-3} m^2/kg)air}$$

$$\dot{D}m = 0.00085 Gy/h$$

Energy absorption (Dose) in a thin slab



$$\psi = \psi_0 e^{-\mu_{ab}x}$$

$$if\ \mu_{ab}x << 1\quad then$$

$$e^{-\mu_{ab}x} \approx 1 - \mu_{ab}x \quad and$$

$$\psi = \psi_0(1 - \mu_{ab}x)$$

$$\psi_0 - \psi = \psi_0\mu_{ab}x$$

rate of energy absorption over an area A

$$(\psi_0 - \psi)A = \psi_0\mu_{ab}x\,A$$

The mass of slab is $m = \rho A x$,

and the dose rate is

$$\dot{D} = \frac{\psi_0\mu_{ab}x\,A}{\rho A x} = \psi_0(\frac{\mu_{ab}}{\rho})slab$$

Assumptions-
- *thin slab relative to mean-free path of photons,*
- *thin enough so bremsstrahlung escapes,*
- *thick enough so that electrons stop in it*

Useful rule: $1/r^2$ rule

- What if you know ϕ at point r_1?
- Create a relationship for flux $\phi(r_2)$ when $\phi(r_1)$ is known at a distance r_1
- Consider when r_1 is
  - 1 foot
  - 1 meter

$$\phi_1 = \frac{S_0}{4\pi r_1^2}$$

$$\phi_2 = \frac{S_0}{4\pi r_2^2}$$

$$\phi_2 = \frac{\phi_1 r_1^2}{r_2^2}$$

$$\phi_2 = \frac{\phi_1}{r_2^2}\quad for\ r = 1$$

where S_0 is the source strength

Calculating Dose Rate in Air

- Given ϕ_0 point source (isotropic) and the following
  - Photon energy E_0 (mono-energetic)
  - No air attenuation
  - ***Mass-energy absorption*** coefficient for air ($\mu_{ab}\ \rho^{-1}$)
- The dose rate is

Where this is **photon flux** [photons/cm^2/s]

$$\dot{D} = \frac{S_0}{4\pi r^2} E \left(\frac{\mu_{ab}}{\rho}\right)$$

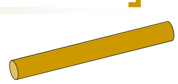
Calculating Dose Equivalent Rate in Air

$$\dot{H}_o = \left(\frac{S}{4\pi r^2}\right)\left(\frac{\mu_{en}}{\rho}\right) E_{gamma}$$

- H_o = total gamma dose equivalent rate – for single gamma emitting source, since w_R =1 for photons
- S is source strength (gamma/s)
- μ_{en}/ρ mass energy absorption coefficient (cm^2 /g)
  - ρ = density (g/cm^3)
- E_{gamma} is a weighted energy of the emitted gamma ray E_{gamma} = E*f*
  - *f* = yield of gamma ray (e.g., fractional abundance)

Until now we were considering a point gamma radiation source!! But…

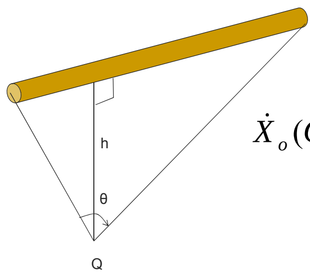
Line Source Geometry



- Examples – resin columns, fuel rods, fluid line, pipe – all can be estimated using line source calculations
- Unattenuated radiation field in air from a line source of length L is:

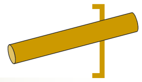
$$\dot{X}_o(Q) = \Gamma \frac{C_L \theta}{h}$$

Line Source Geometry



$$\dot{X}_o(Q) = \Gamma \frac{C_L \theta}{h}$$

Line Source Geometry

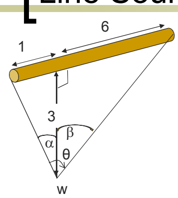


$$\dot{X}_o(Q) = \Gamma \frac{C_L \theta}{h}$$

where

- $X_o(Q)$ = unattenuated
  - exposure rate (R/h), or
  - Dose-equivalent rate (rem/h) at point Q
- Γ is a source strength
- *h* is a perpendicular distance of the point Q to the line source
- A is a total activity (Ci)
- θ is an angle subtended by the line source at the point of interest (radians)
- C_L = source activity per unit length of the line source
  - A/L (Ci/m, Ci/ft, Ci/cm, etc.)

Line Source Geometry



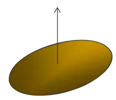
- Determining θ
- The angle θ is obtained from two triangles
- These have a base / height / included angle of
  - 1.0m / 3.0 m / α and
  - 6.0 m / 3.0m / β, respectively
- The total angle subtended by the line source θ is the sum of the angles α and β. Each is determined from its respective tangent functions:
- Tan α = 1/3 Tan β = 6/3
- $\alpha = \tan^{-1}(1/3)$ $\beta = \tan^{-1}(6/3)$
- α = 18.4º (0.322 rad) β = 63.4º (1.11 rad)
- Therefore θ is
  - 18.4 + 63.4 = 81.8º or 1.43 radians

Notice when using this equation, typically the pipe diameter is ignored, which means it is a very thin pipe.

Let's see a problem…

Plain (Disk) Source Geometry

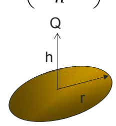
- Examples: spill of radioactive liquid, the radiation field above a resin bed, contaminated surface sources



$$\dot{X}_o(Q) = \pi \Gamma C_a \ln\left(\frac{r^2+h^2}{h^2}\right)$$

Plain Source Geometry

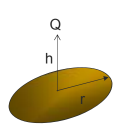
$$\dot{X}_o(Q) = \pi G C_a \ln\left(\frac{r^2+h^2}{h^2}\right)$$



- where
- $X_o(Q)$ = unattenuated
  - exposure rate (R/h), or
  - Dose-equivalent rate (rem/h) at point Q lying on the axis of the disk
- Q is a point at which the dose is determined (it is at the perpendicular distance h on the axis of the disk source)
- A is a total activity (Ci)
- C_a = source surface activity A(Ci) per unit area a
  - A/a (Ci/m², Ci/ft², Ci/cm², etc.)
  - r = radius of the disk source

Plain Source Geometry

$$\dot{X}_o(Q) = \pi G C_a \ln\left(\frac{r^2+h^2}{h^2}\right)$$

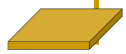


Reminder:

$$\dot{X}_o(Q) = \pi\, G\left(\frac{R \cdot m^2}{Ci \cdot h}\right) C_a \left(\frac{Ci}{m^2}\right) \ln\left(\frac{r^2+h^2}{h^2}\right) (=) R/h$$

$$\dot{X}_o(Q) = 34\pi\, G\left(\frac{C/kg \cdot m^2}{MBq \cdot h}\right) C_a \left(\frac{MBq}{m^2}\right) \ln\left(\frac{r^2+h^2}{h^2}\right) (=) Sv/h$$

Slab Source Geometry

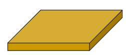


$$\dot{X}_o(h) = \pi \Gamma \frac{C_v}{u}\left(1 - e^{-ut}\right)\ln\left(\frac{R^2 + h^2}{h^2}\right)$$

- Can apply to sources such as contaminated slabs, tanks of contaminated material;
- Whether or not to use a slab or a disk source depends on how accurate you need to be, how far you are away, the concentration of radionuclide, distribution and content.

Slab Source Geometry

$$\dot{X}_o(h) = \pi \Gamma \frac{C_v}{u}\left(1 - e^{-ut}\right)\ln\left(\frac{R^2 + h^2}{h^2}\right)$$



- $X_o(Q)$ = unattenuated
  - exposure rate (R/h), or
  - Dose-equivalent rate (rem/h) at point Z lying on the axis of the slab at a distance h from the surface
- Z =point at which the dose is determined (it is at the perpendicular distance h on the axis of the slab)
- A = total activity (Ci)
- C_v = source activity per unit volume of the line source
  - A/v (Ci/m^3, Ci/ft^3, Ci/cm^3, etc.)
- v = volume of the source (cm^3)
  - = $\pi R^2 t$
  - t = thickness (cm)
  - u = attenuation coefficient (cm^{-1})

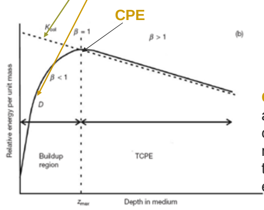
Absorbed Dose – Kerma relationship

- Absorbed dose, D, is the energy imparted by ionizing radiation to the mass Δm: D= ΔE/ Δm.
- **Kerma**, K, is the energy transferred through various processes without correcting for any energy losses after interactions. It is the sum of initial **kinetic energies** of all charged particles liberated by photons in the mass Δm:

$$K = \frac{\Delta Ek}{\Delta m}$$

Absorbed Dose – Kerma relationship

- Absorbed dose is $D = \Phi(\frac{\mu}{\rho})(E_{absorbed})$
- Kerma is $K = \Phi(\frac{\mu}{\rho})(E_{transferred})$



- Kerma continuously decreases due to decrease of the flux of indirectly ionizing electrons;
- Dose increases as the electronic equilibrium is approached, after which it decreases with the depth in medium;

Charged Particle Equilibrium (CPE) exists at point p, centered in a volume V, if each charged particle carrying an energy out of V is replaced by another identical charged particle that carries the same energy into V. If CPE exists as a point D = K.

Average Ion-Pair Energy, W

Definition: average energy (W) required to produce an ion-pair in a medium traversed by electrons.

In air: **W = 33.97** eV / ion pair

Units: eV / ion pair or Joule / C